Principal Formulas in Part I

Notation (Chapter 2)

$$\text{(Measured value of } x) = x_{\text{best}} \pm \delta x, \qquad \text{(p. 13)}$$

where

$$x_{\text{best}} = \text{best estimate for } x,$$
$$\delta x = \text{uncertainty or error in the measurement.}$$

$$\text{Fractional uncertainty} = \frac{\delta x}{|x_{\text{best}}|}. \qquad \text{(p. 28)}$$

Propagation of Uncertainties (Chapter 3)

If various quantities $x, \ldots, w$ are measured with small uncertainties $\delta x, \ldots, \delta w$, and the measured values are used to calculate some quantity q, then the uncertainties in $x, \ldots, w$ cause an uncertainty in q as follows:
If q is the sum and difference, $q = x + \cdots + z - (u + \cdots + w)$, then

$$\delta q \begin{cases} = \sqrt{(\delta x)^2 + \cdots + (\delta z)^2 + (\delta u)^2 + \cdots + (\delta w)^2} \\ \qquad \text{for independent random errors;} \\ \leqslant \delta x + \cdots + \delta z + \delta u + \cdots + \delta w \\ \qquad \text{always.} \end{cases} \qquad \text{(p. 60)}$$

If q is the product and quotient, $q = \dfrac{x \times \cdots \times z}{u \times \cdots \times w}$, then

$$\frac{\delta q}{|q|} \begin{cases} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2} \\ \qquad \text{for independent random errors;} \\ \leqslant \frac{\delta x}{|x|} + \cdots + \frac{\delta z}{|z|} + \frac{\delta u}{|u|} + \cdots + \frac{\delta w}{|w|} \\ \qquad \text{always.} \end{cases} \qquad \text{(p. 61)}$$

If $q = Bx$, where B is known exactly, then

$$\delta q = |B|\,\delta x. \qquad \text{(p. 54)}$$

If q is a function of one variable, $q(x)$, then

$$\delta q = \left|\frac{dq}{dx}\right| \delta x. \qquad \text{(p. 65)}$$

If q is a power, $q = x^n$, then

$$\frac{\delta q}{|q|} = |n| \frac{\delta x}{|x|}. \qquad \text{(p. 66)}$$

If q is any function of several variables $x, \ldots, z$, then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \cdots + \left(\frac{\partial q}{\partial z}\delta z\right)^2} \qquad \text{(p. 75)}$$

(for independent random errors).

Statistical Definitions (Chapter 4)

If $x_1, \ldots, x_N$ denote N separate measurements of one quantity x, then we define:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \text{mean}; \qquad \text{(p. 98)}$$

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum (x_i - \bar{x})^2} = \text{standard deviation, or SD} \qquad \text{(p. 100)}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \text{standard deviation of mean, or SDOM.} \qquad \text{(p. 102)}$$

The Normal Distribution (Chapter 5)

For any limiting distribution $f(x)$ for measurement of a continuous variable x:

$$f(x)\,dx = \text{probability that any one measurement will give an answer between } x \text{ and } x + dx; \qquad \text{(p. 128)}$$

$$\int_a^b f(x)\,dx = \text{probability that any one measurement will give an answer between } x = a \text{ and } x = b; \qquad \text{(p. 128)}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \text{ is the normalization condition.} \qquad \text{(p. 128)}$$

The Gauss or normal distribution is

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}, \qquad \text{(p. 133)}$$

where

$$X = \text{center of distribution} = \text{true value of } x$$
$$= \text{mean after many measurements,}$$
$$\sigma = \text{width of distribution}$$
$$= \text{standard deviation after many measurements.}$$

The probability of a measurement within t standard deviations of X is

$$Prob(\text{within } t\sigma) = \frac{1}{\sqrt{2\pi}}\int_{-t}^{t} e^{-z^2/2}\,dz = \text{normal error integral}; \qquad \text{(p. 136)}$$

in particular

$$Prob(\text{within } 1\sigma) = 68\%.$$