Physics 307L

Spring 2021

Prof. Darcy Barron

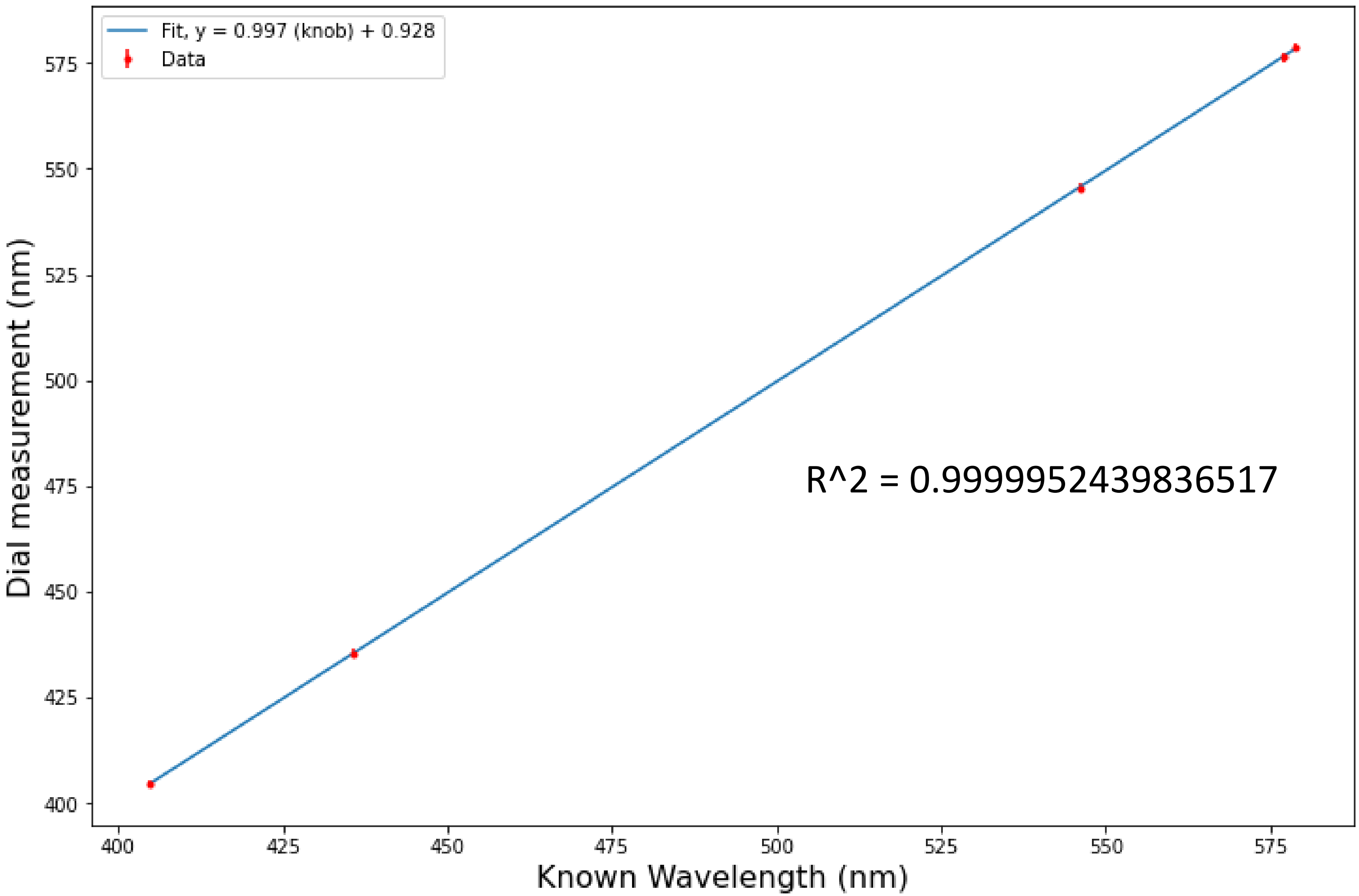
This lecture will be recorded

Updated Schedule

- Schedule of assignment due dates through the end of the semester is now posted on wiki, and will be in Teams soon
  - https://ghz.unm.edu/juniorlab/index.php?title=Schedule_Spring_2021#Course_Schedule
- **First lab report due date is this Wed, March 31**
  - Please ask if you have questions about completing the analysis for the experiment you write about in Lab Report 1
  - Lab report cannot be on Lab 0
  - https://ghz.unm.edu/education/juniorlab_pdfs/labreportguidelines.pdf
- **Schedule for Talk 2 is posted**
- **All students will give Talk 3** during scheduled during final exam time for this class, Friday, May 14 from 12:30pm – 2:30pm
  - This should not conflict with any other scheduled finals, but please let me know if there is a problem

Balmer Series Analysis

- Straightforward steps
  - Estimating uncertainties from equipment
  - Repeating measurements to estimate uncertainty
  - Propagating uncertainties
  - Simple linear fit to data
- More complex steps
  - Rejecting data (Chapter 6 of Taylor)
  - Choosing how to combine separate measurements
  - Least-squares fitting with errors in both dimensions



The slope = 0.9974273274123038, with uncertainty 0.0012558490265354887 The intercept = 0.9282554690792497, with uncertainty 0.6452914621011211

Chauvenet's Criterion

- If you make N measurements of a single quantity x, Chauvenet's criterion gives a simple test for deciding whether to reject a 'suspect value'
- $t_{sus} = \frac{|x_{sus} - \bar{x}|}{\sigma_x}$
- $n = N \times Prob(outside\ t_{sus}\sigma)$
  - Use Appendix A to look up values
- If n < 0.5, then it is reasonable to reject x_{sus}

Example

- We make 10 measurements of length, x, and get these results:
  - 46, 48, 44, 38, 45, 47, 58, 44, 45, 43
  - What is the mean of the dataset? **45.8**
  - What is the standard deviation? **5.1**
  - What is the suspicious value? **58**
  - What is probability that such an outlier would appear from random chance?
    - **0.016**

Example 2

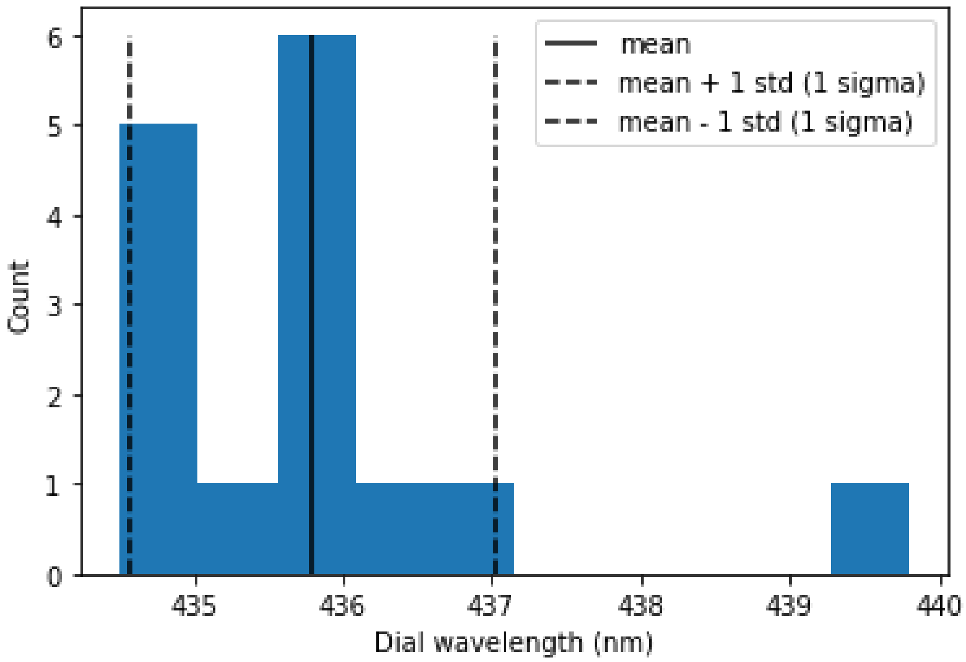
- Multiple measurements of second shortest wavelength line of mercury emission
- `[437, 435.0, 435, 434.5, 439.8, 435.4, 435.6, 435.7, 436.2, 435.7, 436.0, 435.7, 435.8, 434.8, 434.8]`
- What is mean? What is standard deviation?
  - **435.8, 1.23**
- What value is suspect? **439.8**
- Should it be thrown out and why?

Example 2

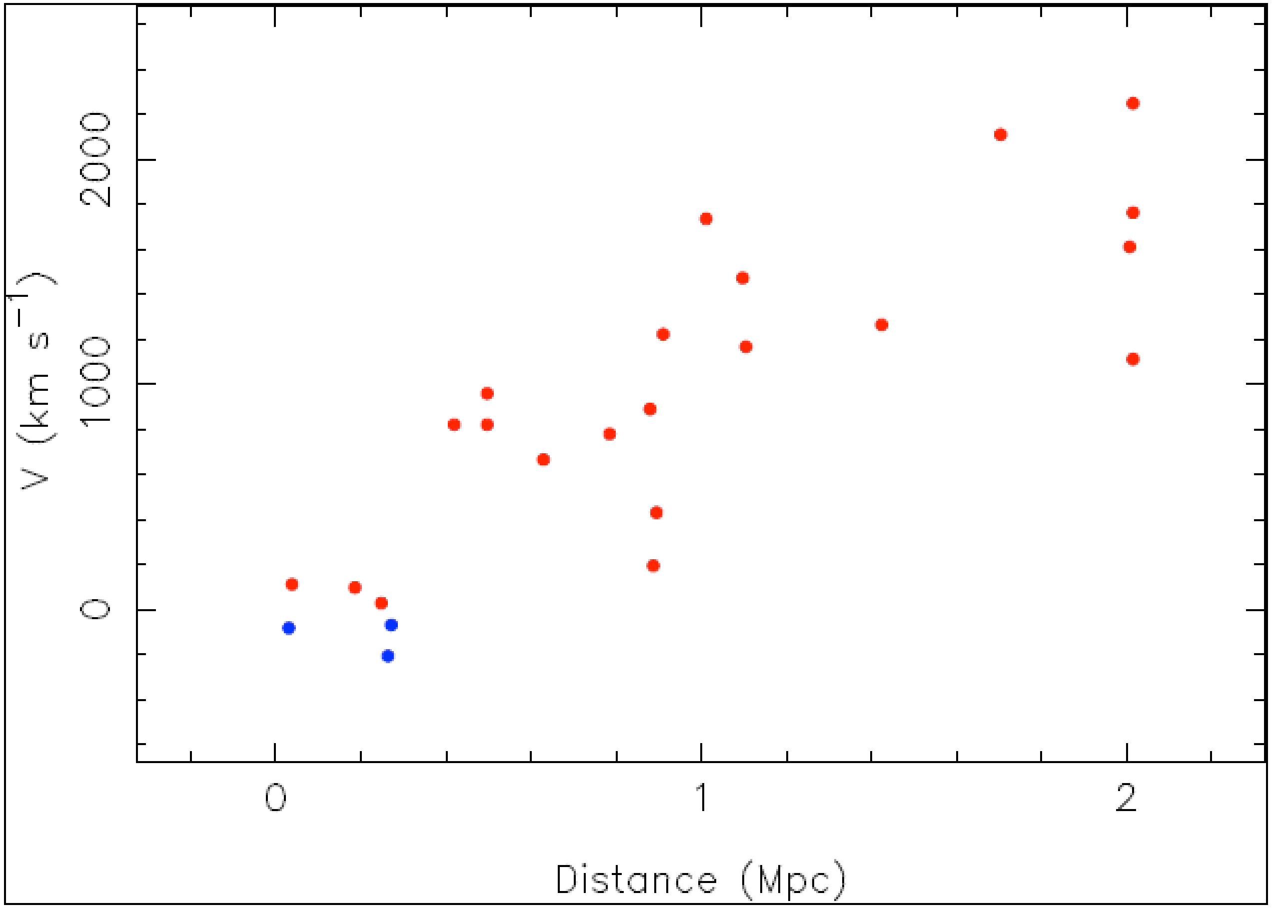
- Multiple measurements of second shortest wavelength line of mercury emission
- `[437, 435.0, 435, 434.5, 439.8, 435.4, 435.6, 435.7, 436.2, 435.7, 436.0, 435.7, 435.8, 434.8, 434.8]`
- Should it be thrown out and why?
  - Probably!
  - t-score is 3.24, so this is a 3-sigma outlier
  - 99.73% of random fluctuations will fall within 3 sigma of mean
  - So, there is a less than 0.27% chance one value would fluctuate this far away
  - We have 15 values, so there's a 4% chance it could have happened in this dataset
  - There is reasonable suspicion that the value might have been written down incorrectly or some other fluke, so it seems ok to throw it out

Example 2

- Multiple measurements of second shortest wavelength line of mercury emission
- `[437, 435.0, 435, 434.5, 439.8, 435.4, 435.6, 435.7, 436.2, 435.7, 436.0, 435.7, 435.8, 434.8, 434.8]`



Fitting data to a model



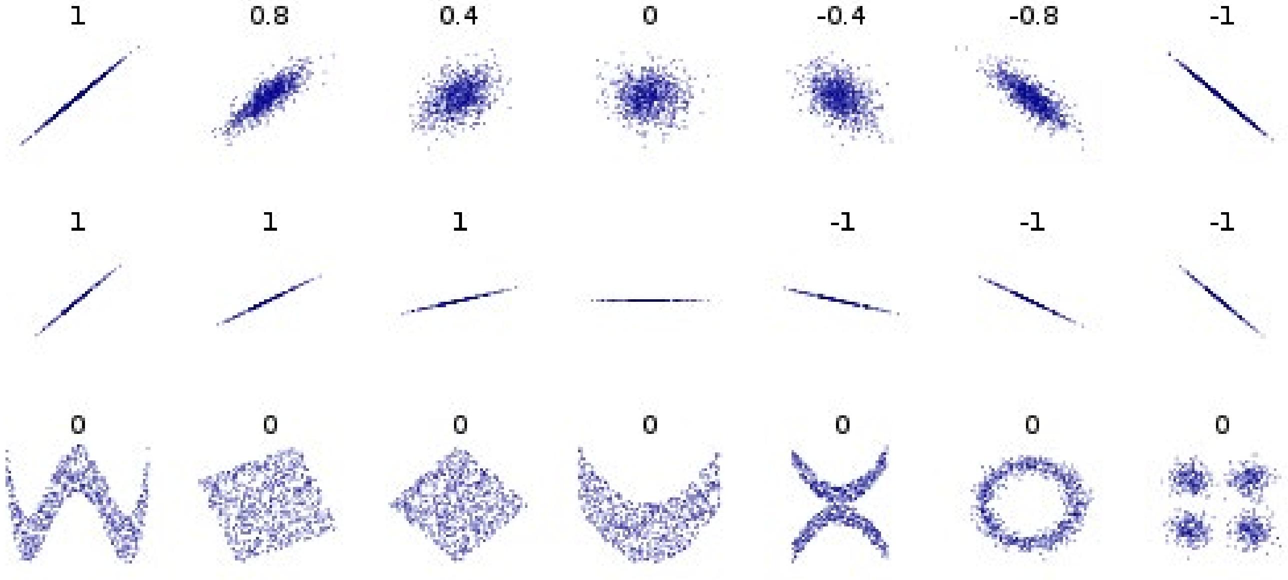
An early Hubble diagram, N=24 galaxies (1936)

Simplifying correlations

- **Linear correlations are easy to plot and examine**
- Linear fit
    - y=mx+b
- Can linearize your data to make it a linear correlation
    - Example: Balmer Series
        - $1/\lambda = R\ [\ 1/2^2 - 1\ /n^2]$
        - Set $y = 1/\lambda$
- Proportional correlation is special case of linear fit where y-intercept is fixed at zero
    - y = mx

Pearson’s correlation coefficient

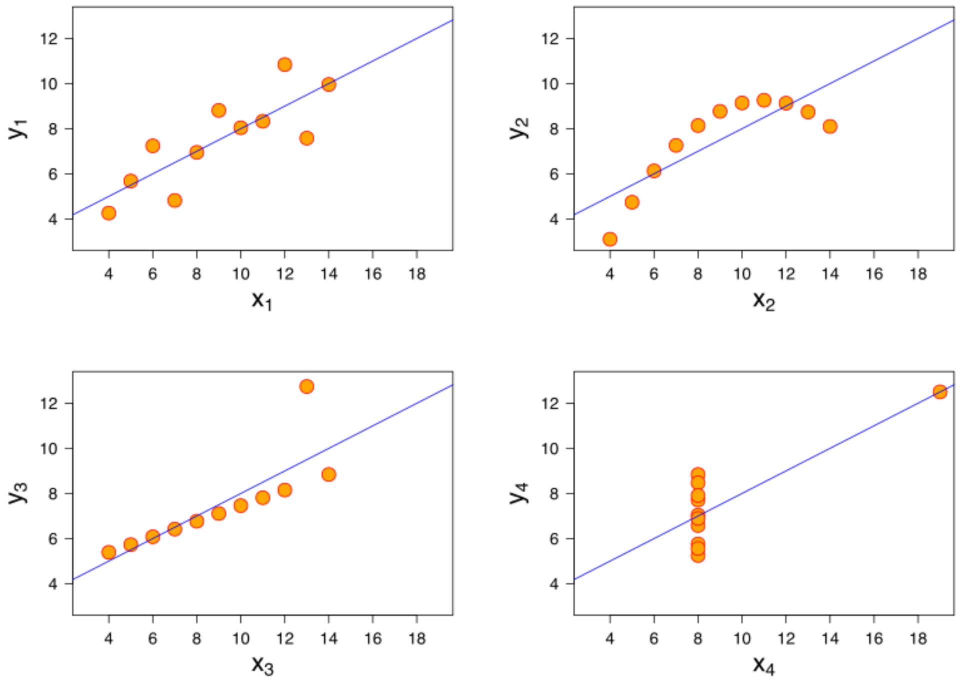
https://en.wikipedia.org/wiki/Pearson_correlation_coefficient



- Pearson’s correlation coefficient, r, measures the linear correlation between two variables

Linear correlations are easy to plot and examine

Anscombe's quartet



Property	Value	Accuracy
Mean of *x*	9	exact
Sample variance of *x*	11	exact
Mean of *y*	7.50	to 2 decimal places
Sample variance of *y*	4.125	±0.003
Correlation between *x* and *y*	0.816	to 3 decimal places
Linear regression line	$y = 3.00 + 0.500x$	to 2 and 3 decimal places, respectively
Coefficient of determination of the linear regression	0.67	to 2 decimal places

Plotting data can provide insight on trends not captured by statistics

https://en.wikipedia.org/wiki/Anscombe%27s_quartet

Speed of Light Analysis

- For our setup measuring the speed of light, we strongly expect to get data that fits a straight line
- For most ways of recording data, the data follows a linear fit with $y = mx + b$
- For some ways of recording data, it is a proportional fit $y = mx$
- What is the meaning of the slope?
- What is the meaning of the y-intercept?

Speed of light experimental design

- Without recording any data, we can say a few things about the capabilities and limitations of the experiment
  - We are performing the experiment on an optics table, so our maximum length is of order 1 meter, and our uncertainty is of order 0.5 cm
    - Placement error gets multiplied by 2 because roundtrip
  - We are using a fast pulsed laser and a fast oscilloscope, which sets our time resolution to ~ 0.5 ns
    - Are we limited by the laser, or the oscilloscope, or…?
    - How well calibrated is the oscilloscope?
- Expect ~ 5-10% in systematic errors in our measurement of the speed of light in one leg of the optical path, limited by uncertainties in time measurement
- Can measure time delay added by adding something to optical path (acrylic rod), and derive index of refraction (with length)

Least-squares fitting

- Procedure described in Taylor Chapter 8
- Fitting multiple data points to a line can tell us how well the data is described by a linear fit, and gives a measure of statistical (random) fluctuations in our data set, or any lurking remaining systematic errors
- Just like with statistics (mean, standard deviation), knowing how it is done is important, so good to do it 'by hand' a few times
- However, also important to learn (*and understand*) at least one way to do it through programming
  - Excel, Python, Matlab, etc.
  - https://ghz.unm.edu/juniorlab/index.php?title=ErrorAnalysis
  - Can get into many variations once you start to weight data (touched on in Chapter 7 – weighted averages)

Example in Taylor

Table 8.1. Masses m_i (in kg) and lengths l_i (in cm) for a spring balance. The "x" and "y" in quotes indicate which variables play the roles of x and y in this example.

Trial number i	"x" Load, m_i	"y" Length, l_i	m_i^2	$m_i l_i$
1	2	42.0	4	84
2	4	48.4	16	194
3	6	51.3	36	308
4	8	56.3	64	450
5	10	58.6	100	586
$N = 5$	$\Sigma m_i = 30$	$\Sigma l_i = 256.6$	$\Sigma m_i^2 = 220$	$\Sigma m_i l_i = 1{,}622$

Example in Taylor

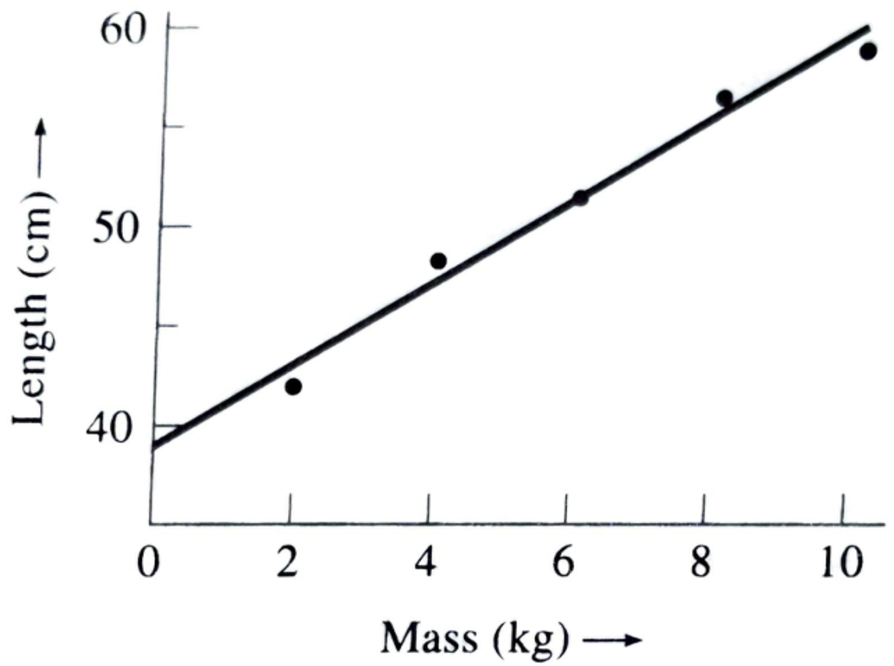
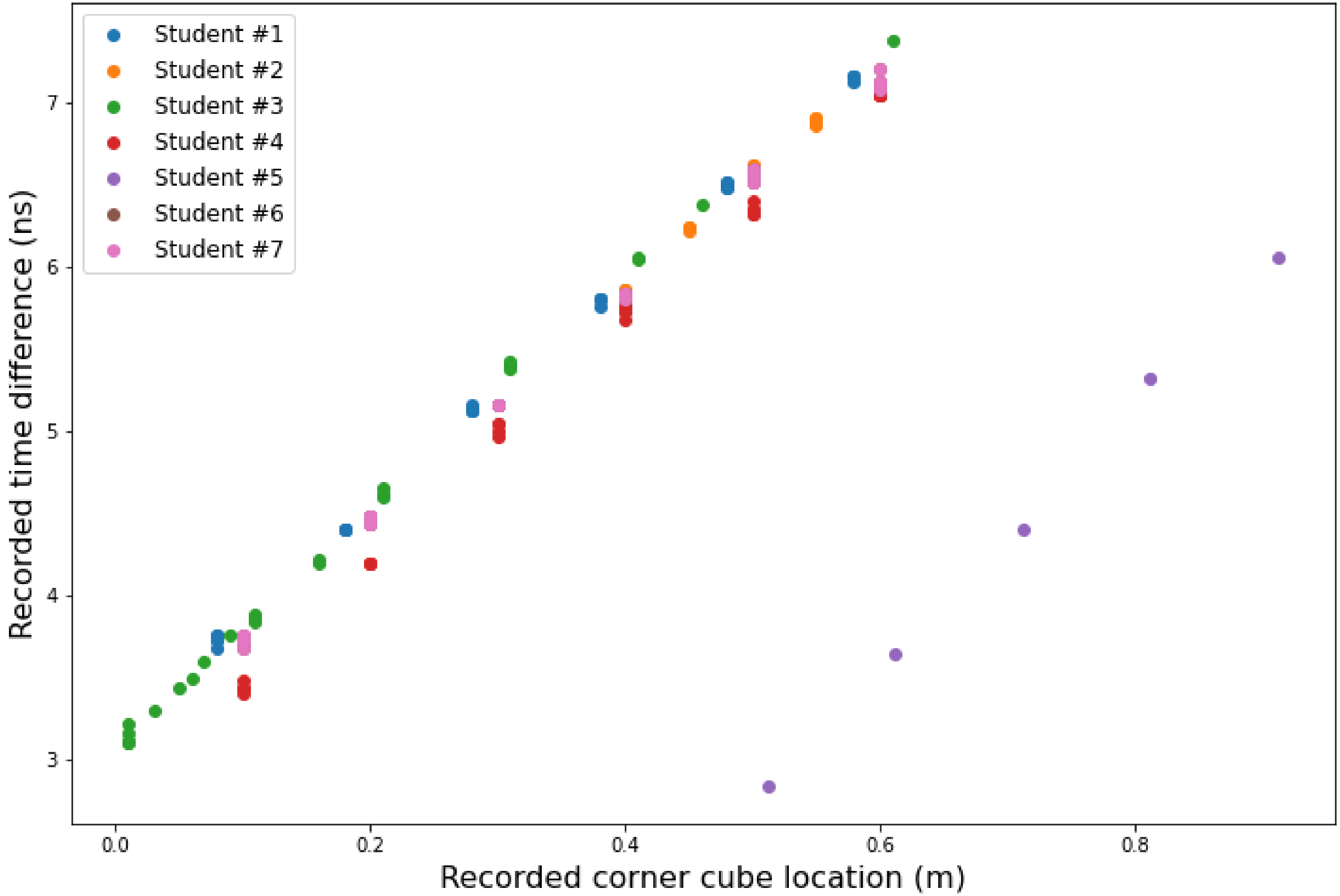
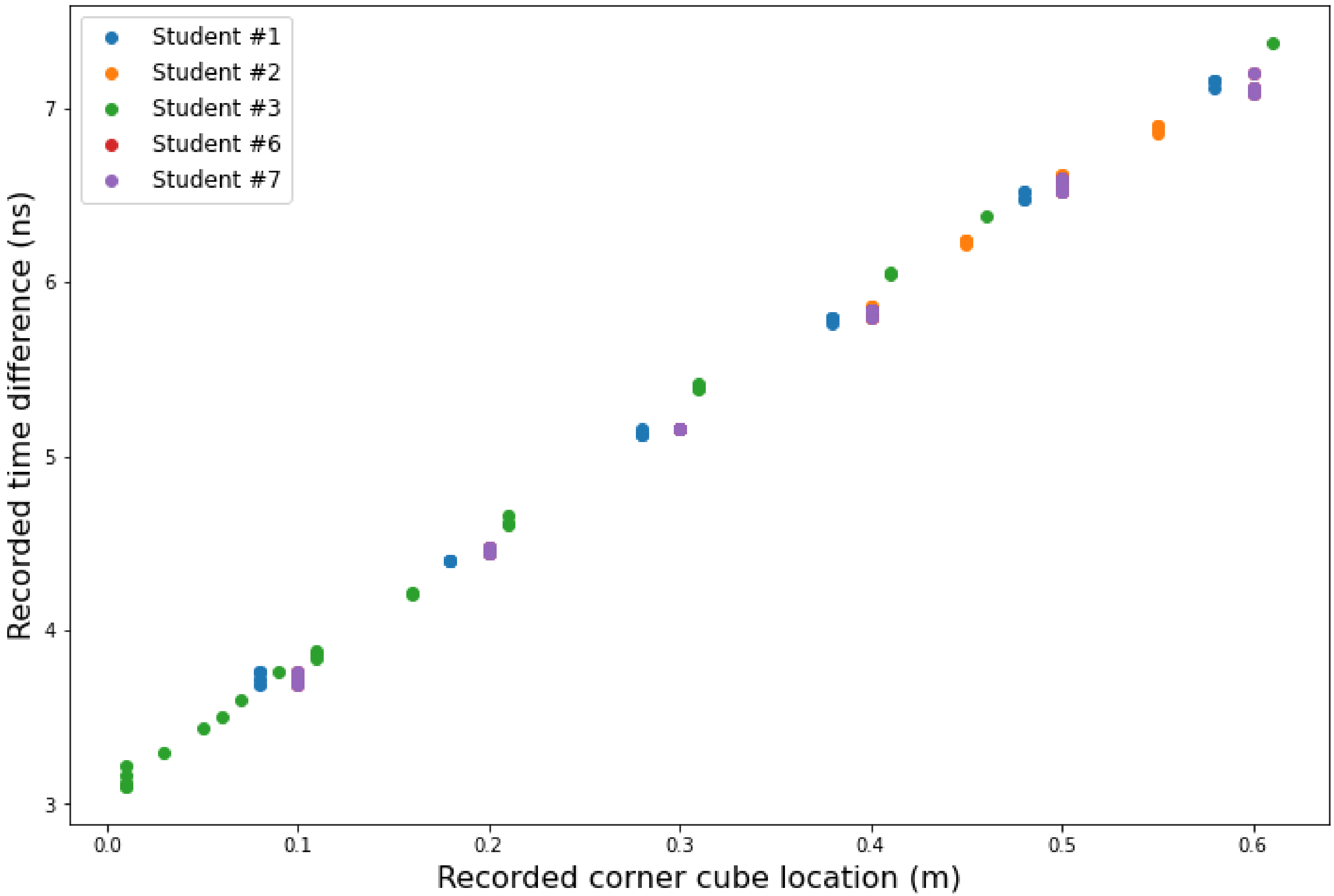


Figure 8.2. A plot of the data from Table 8.1 and the best-fit line (8.13).

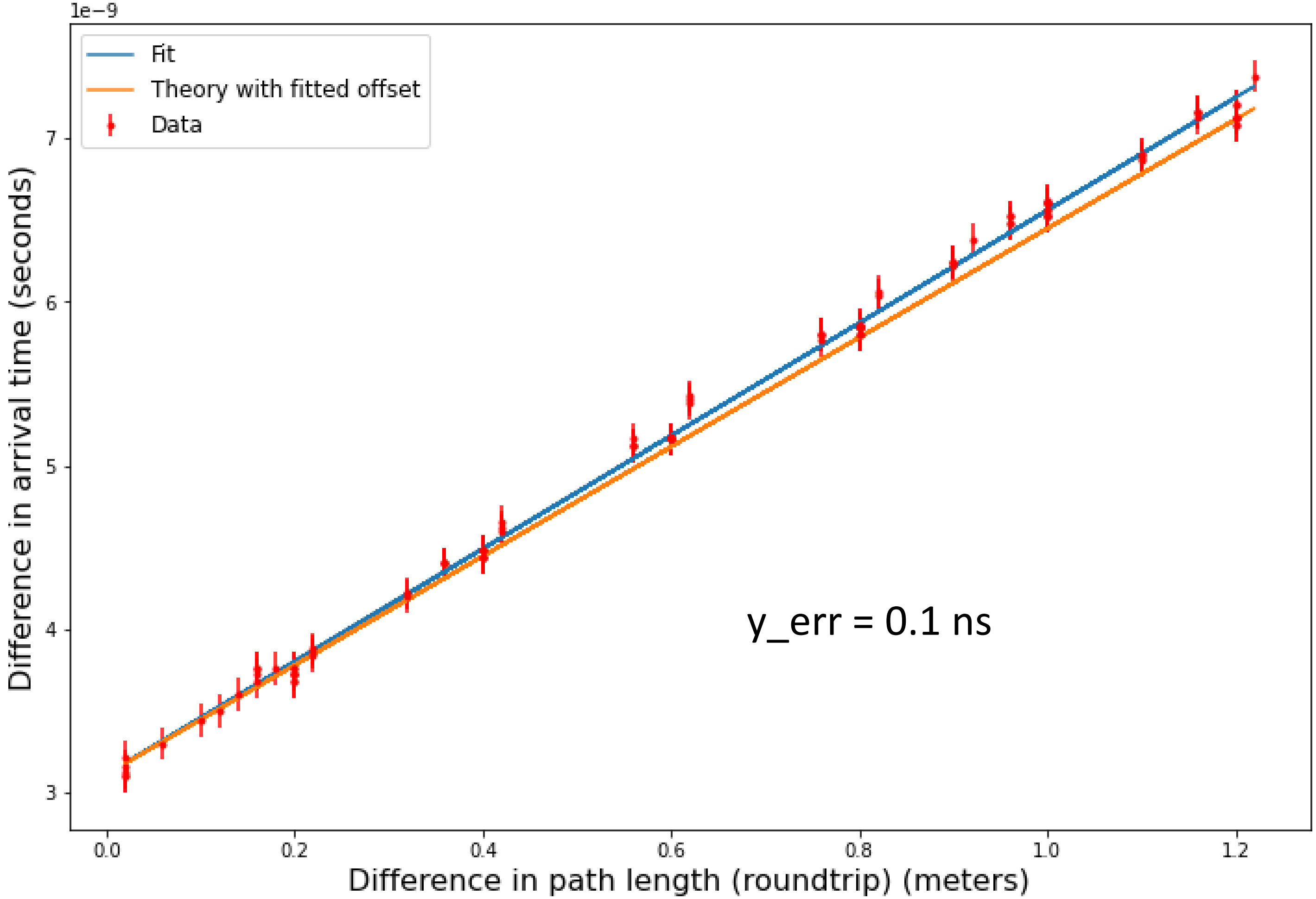
Speed of Light Data – Our Class



Speed of Light Data – Our Class Scrubbed of Inconsistent Data



Speed of Light Data – Our Class
Best fit to data from 5 students



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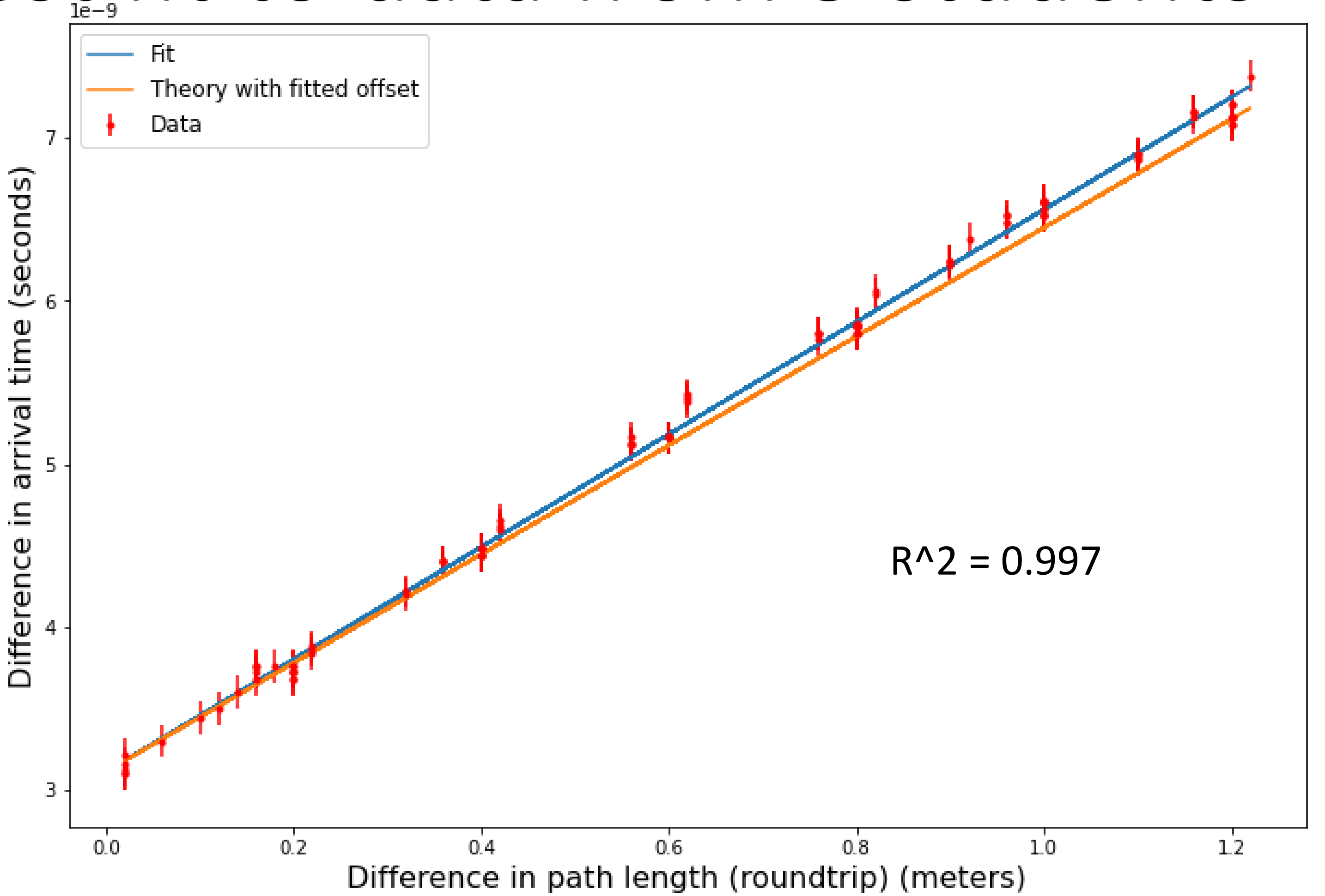


```
The slope = 3.4470692242258648e-09, with uncertainty 1.741025266191759e-11
The intercept = 3.111703798698238e-09, with uncertainty 1.298000077865901e-11
The speed of light = 290101513.76480633
```

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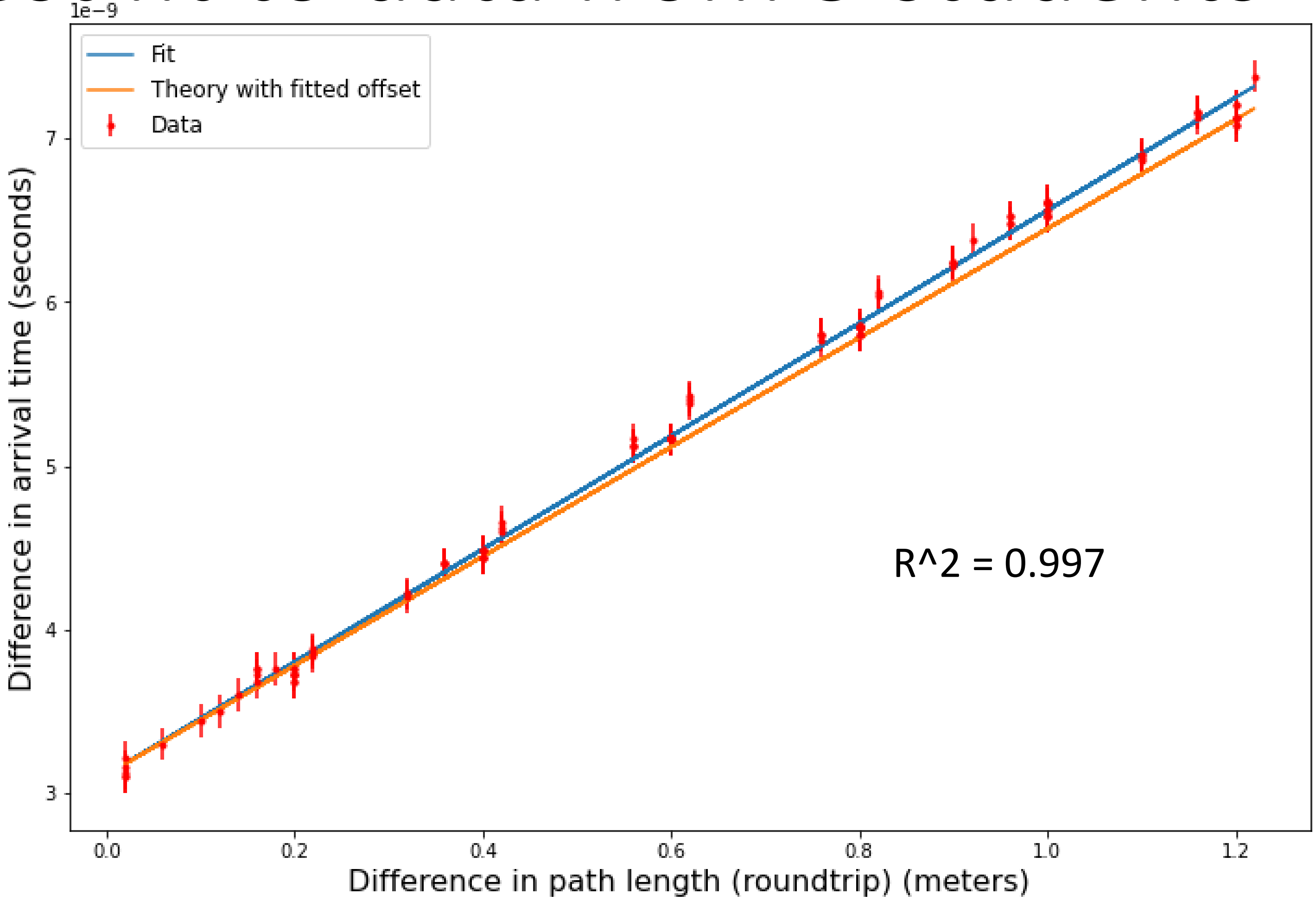


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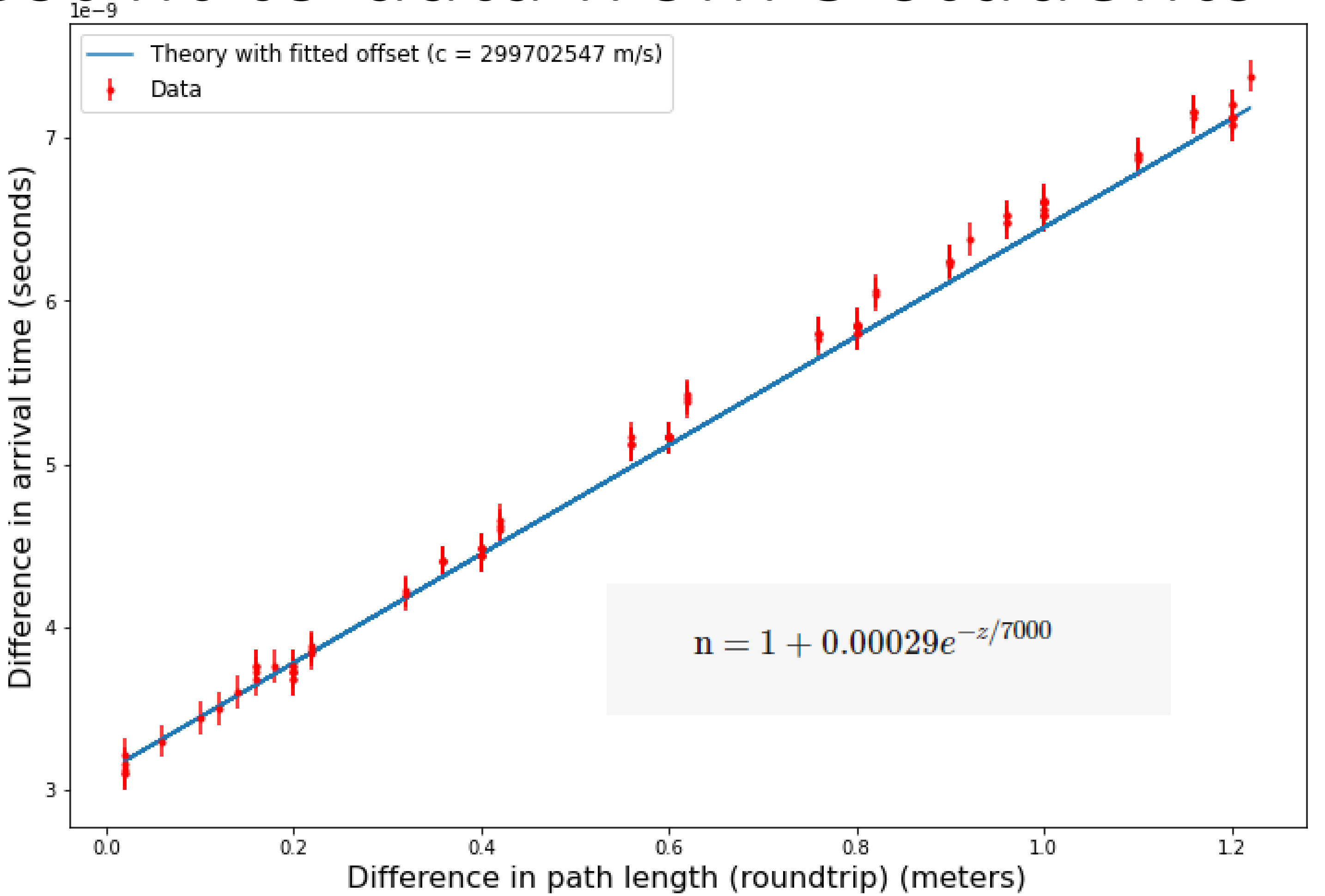

Can we reject the hypothesis that the speed of light is 299,700,000 m/s in our lab?

Examples of student notebooks

2.60

$x_0 = 0$ $\Delta t_0 = 2.0$ns

Trial 1

Δx	$\Delta(\Delta t)$
10.0cm	0.84ns
20.0cm	0.16ns ← This was actually 1.60 and was a typo
30.0cm	2.36ns
40.0cm	3.08ns
50.0cm	3.8ns
60.0cm	4.49ns

$x_0 = 0$ $\Delta t_0 = 2.64$ns

Trial 2

Δx	$\Delta(\Delta t)$
10.0cm	0.76ns
20.0cm	1.56ns
30.0cm	2.40ns
40.0cm	3.08ns
50.0cm	3.68ns
60.0cm	4.40ns

$x_0 = 0$ $\Delta t_0 = 2.60$ns

Trial 3

Δx	$\Delta(\Delta t)$
10.0cm	0.80ns (.8)
20.0cm	1.6ns
30.0cm	2.44ns
40.0cm	3.16ns
50.0cm	3.72ns
60.0cm	4.44ns

2.60

$x_0 = 0$, $\Delta t_0 = 2.0$ns

Trial 4

Δx	$\Delta(\Delta t)$	
10.0cm	~~2.0ns~~	0.84ns
20.0cm	1.60ns	
30.0cm	2.40ns	
40.0cm	3.16ns	
50.0cm	3.76ns	
60.0cm	4.44ns	

corner mirror distance	51.25cm	61.25cm	71.25cm	81.25cm	91.25cm
temporal seperation	2.84ns	3.64ns	4.4ns	5.32ns	6.06ns

For x the position of the corner-cube reflector and t the pulse delay:

The added distance to the light beam is roughly 2(x+5)cm with 5cm the distance from the beam splitter to the mirror in the setup

Data

		Trial 1	Trial 2	Trial 3	Trial 4	Mean
x (m)	2x+0.1 (m)	t (ns)	t (ns)	t (ns)	t (ns)	t (ns)
0.58	1.26	7.16	7.12	7.16	7.16	7.15
0.48	1.06	6.48	6.52	6.52	6.48	6.5
0.38	0.86	5.8	5.76	5.8	5.8	5.79
0.28	0.66	5.16	5.12	5.12	5.12	5.13
0.18	0.46	4.4	4.4	4.4	4.4	4.4
0.08	0.26	3.76	3.68	3.72	3.76	3.73

I varied the separation of the corner cube by 5 cm every measurement. Recorded below are the measurements I got at each distance.

Corner cube location	Δt trial 1	Δt trial 2	Δt trial 3	Δt trial 4
55 cm	6.90 ns ± 0.02 ns	6.86 ns ± 0.02 ns	6.90 ns ± 0.02 ns	6.88 ns ± 0.02 ns
50 cm	6.62 ns ± 0.02 ns	6.60 ns ± 0.02 ns	6.62 ns ± 0.02 ns	6.60 ns ± 0.02 ns
45 cm	6.24 ns ± 0.02 ns	6.24 ns ± 0.02 ns	6.24 ns ± 0.02 ns	6.22 ns ± 0.02 ns
40 cm	5.80 ns ± 0.02 ns	5.86 ns ± 0.02 ns	5.86 ns ± 0.02 ns	5.84 ns ± 0.02 ns

Set 1

Position 1: $x = 61.0 \pm 0.1$ cm

$t = 7.12 \pm 0.01$ ns (Air)

$t = 7.56 \pm 0.01$ ns (rod)

Position 2: $x = 52.0 \pm 0.1$ cm

$t = 6.60 \pm 0.01$ ns (Air) $t = 6.92 \pm 0.01$ ns (rod)

Position 3: $x = 44.0 \pm 0.1$ cm

$t = 6.12 \pm 0.01$ ns (Air) $t = 6.52 \pm 0.01$ ns (rod)

Position 4: $x = 37.0 \pm 0.1$ cm

$t = 5.52 \pm 0.01$ ns (Air) $t = 5.96 \pm 0.01$ ns (rod)

Position 5: $x = 30.0 \pm 0.1$ cm

$t = 5.12 \pm 0.01$ ns (Air) $t = 5.52 \pm 0.01$ ns (rod)

Set 2

Position 1: $x = 55.0 \pm 0.1$ cm

$t = 6.80 \pm 0.01$ ns (Air) $T = 7.24 \pm 0.01$ ns (rod)

Position 2: $x = 50.0 \pm 0.1$ cm

$t = 6.48 \pm 0.01$ ns (Air) $t = 6.89 \pm 0.01$ ns (rod)

Position 3: $x = 45.0 \pm 0.1$ cm

$t = 6.20 \pm 0.01$ ns (Air) $t = 6.52 \pm 0.01$ ns (rod)

Position 4: $x = 40.0 \pm 0.1$ cm

$t = 5.80 \pm 0.01$ ns (Air) $t = 6.16 \pm 0.01$ ns (rod)

Position 5: $x = 35.0 \pm 0.1$ cm

$t = 5.52 \pm 0.01$ ns (Air) $t = 5.80 \pm 0.01$ ns (rod)