Physics 307L

Spring 2021

Prof. Darcy Barron

Lecture 3: Error Analysis

Course webpage

Working on significant updates to course wiki:
https://ghz.unm.edu/juniorlab/

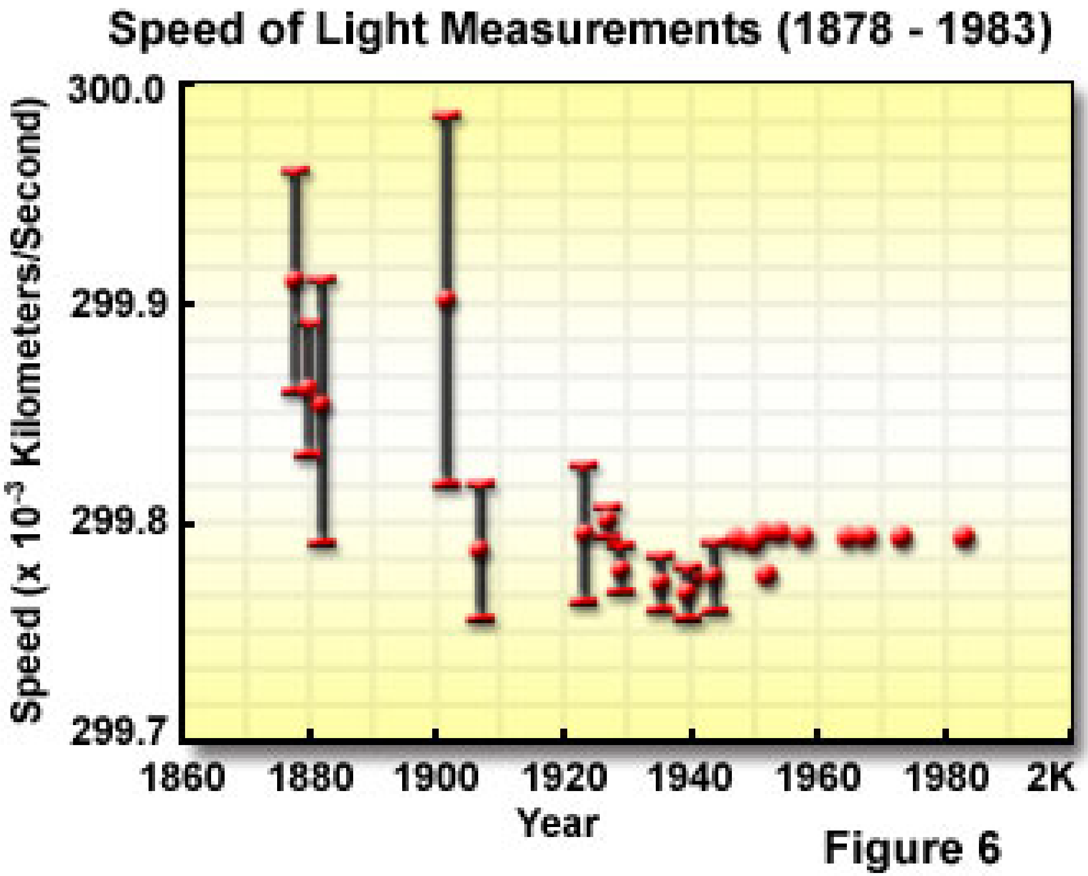
Please check your email regularly for course announcements and updates.

Last time: keeping a lab notebook

- To communicate to the instructors how you performed an experiment and what result you got (and why)
- To document for yourself what you did, especially between lab sessions
  - **Need to include enough details to complete data analysis outside of the lab**
- To practice scientific documentation

- First two lab notebooks are being graded this week, and will be graded on a curve
- Future lab notebooks will need to meet guidelines in rubric for full credit

History of the Speed of Light



https://www.olympus-lifescience.com/en/microscope-resource/primer/lightandcolor/speedoflight/

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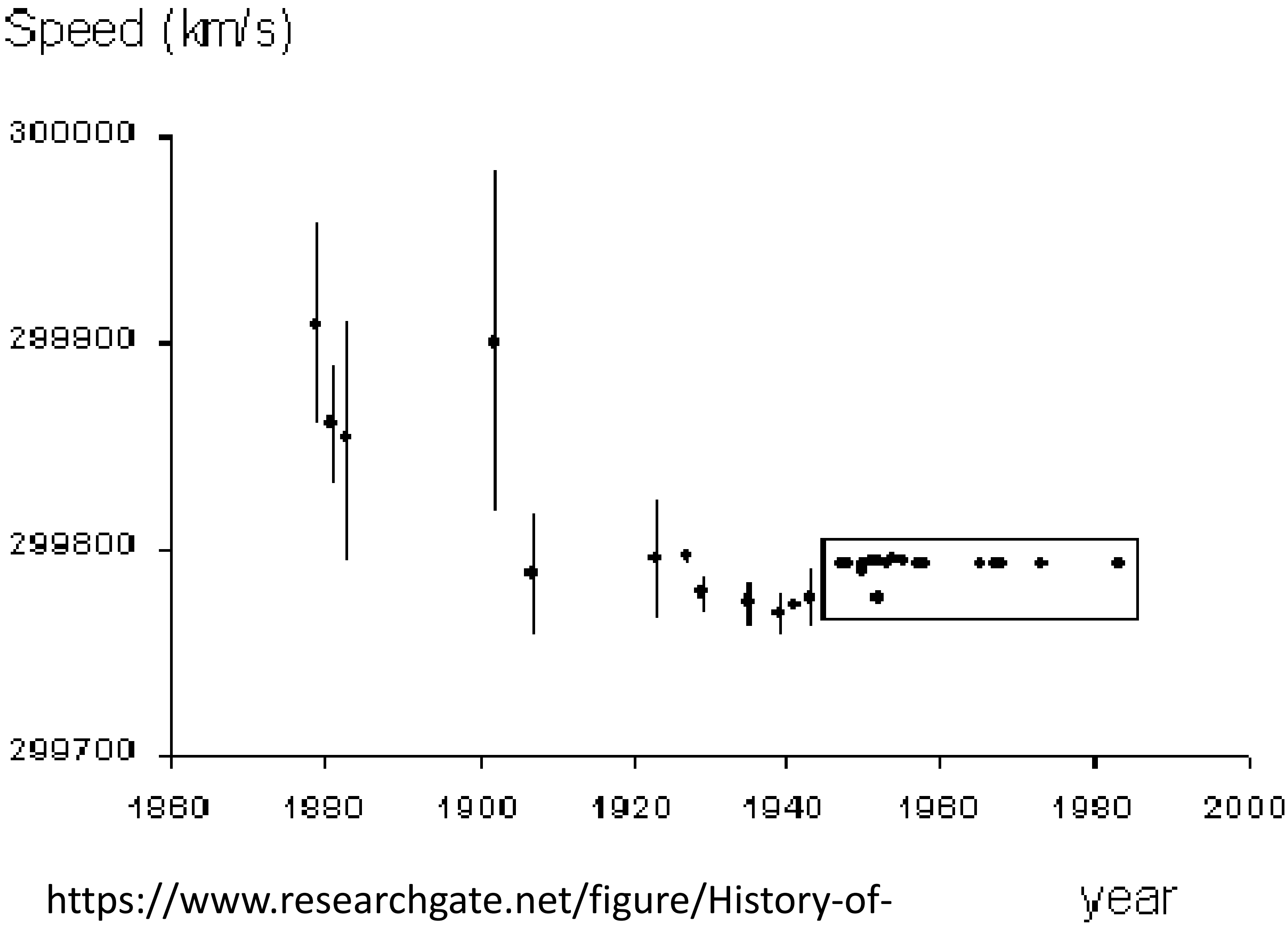

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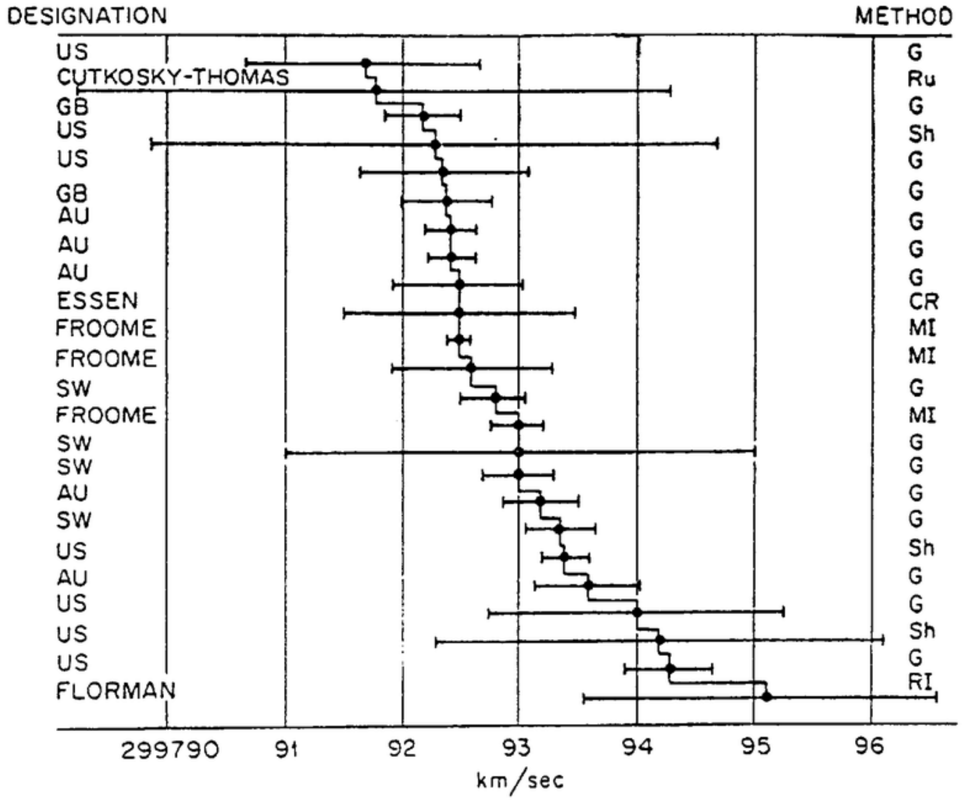


https://www.researchgate.net/figure/Measurements-of-the-speed-of-light-with-the-reported-errors-from-Youden-1972-giving-as_fig2_269942745

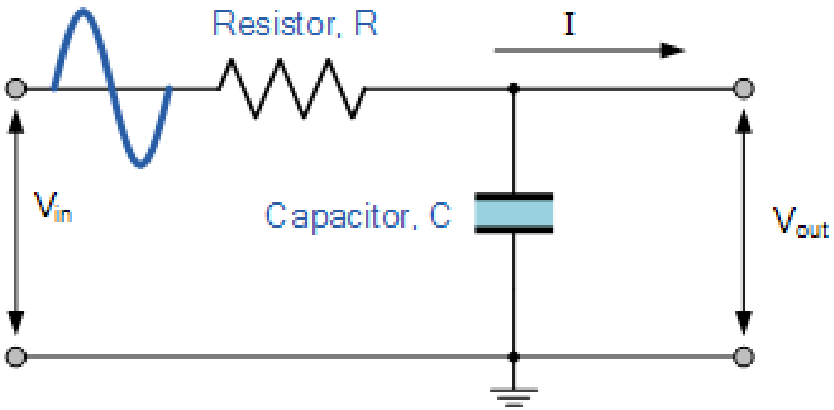
Why do we need ERROR ANALYSIS?

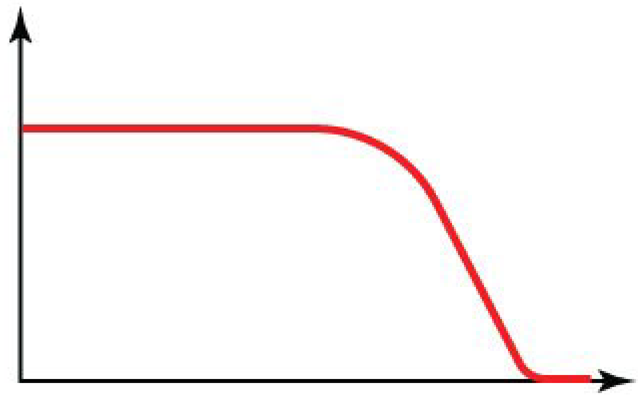
Experimental results are only *ESTIMATES*

This is due to:

- **Uncertainties**
- **Randomness**
- **Limits of precision**
- **Equipment limitations**
- **Incomplete physical model**

Low-pass RC filter





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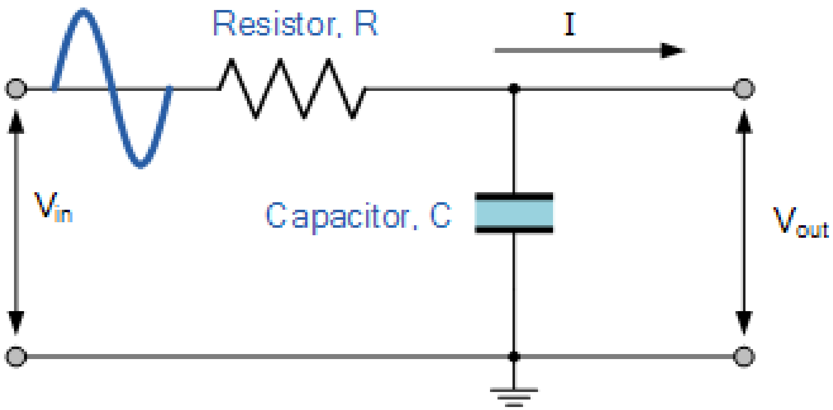

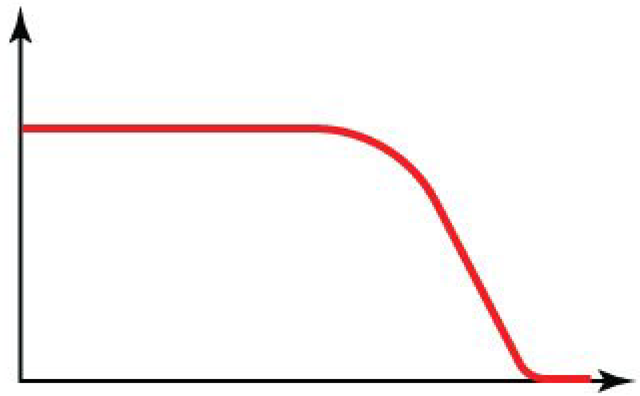

$$\tau = RC = \frac{1}{2\pi fc}$$

$$fc = \frac{1}{2\pi RC} \text{ or } \frac{1}{2\pi\tau}$$

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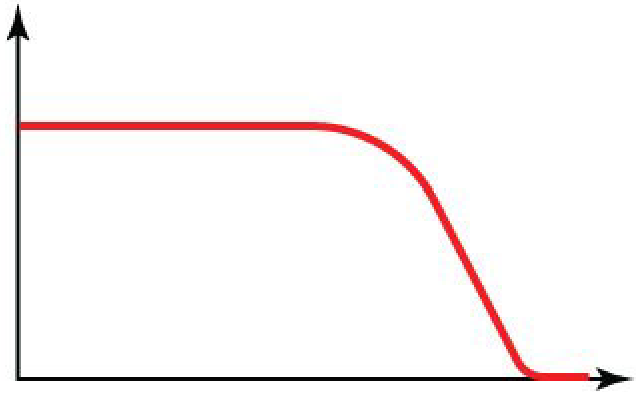

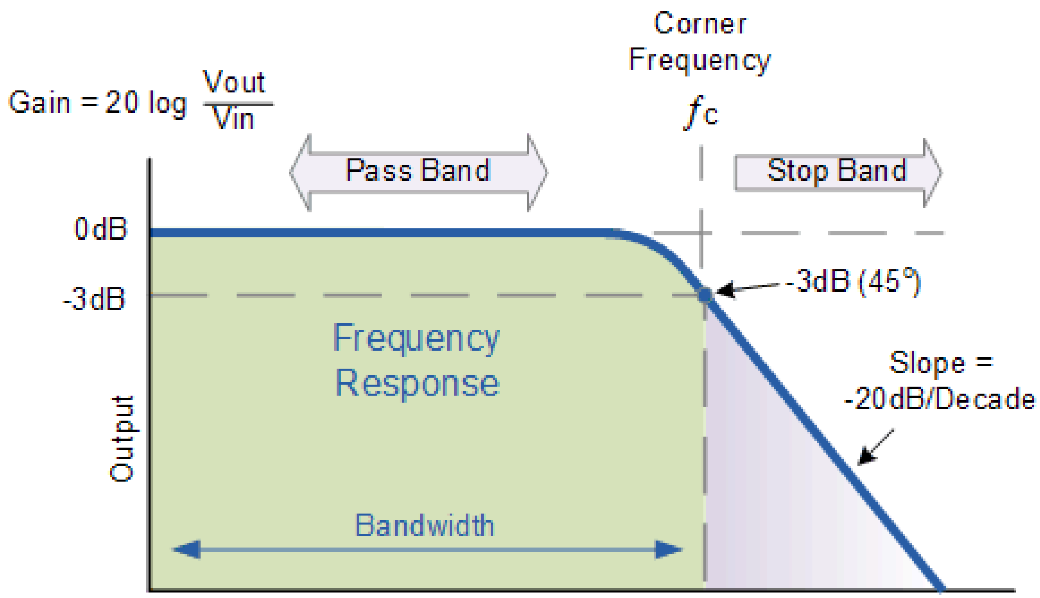

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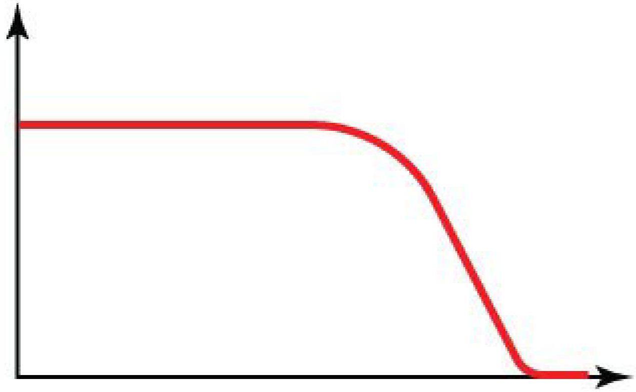

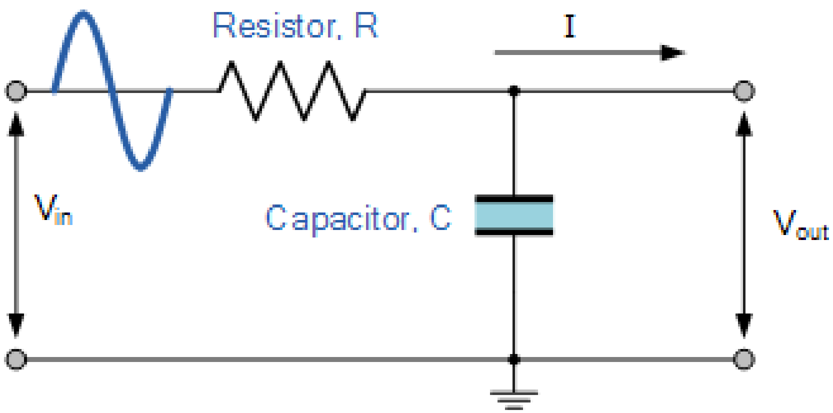

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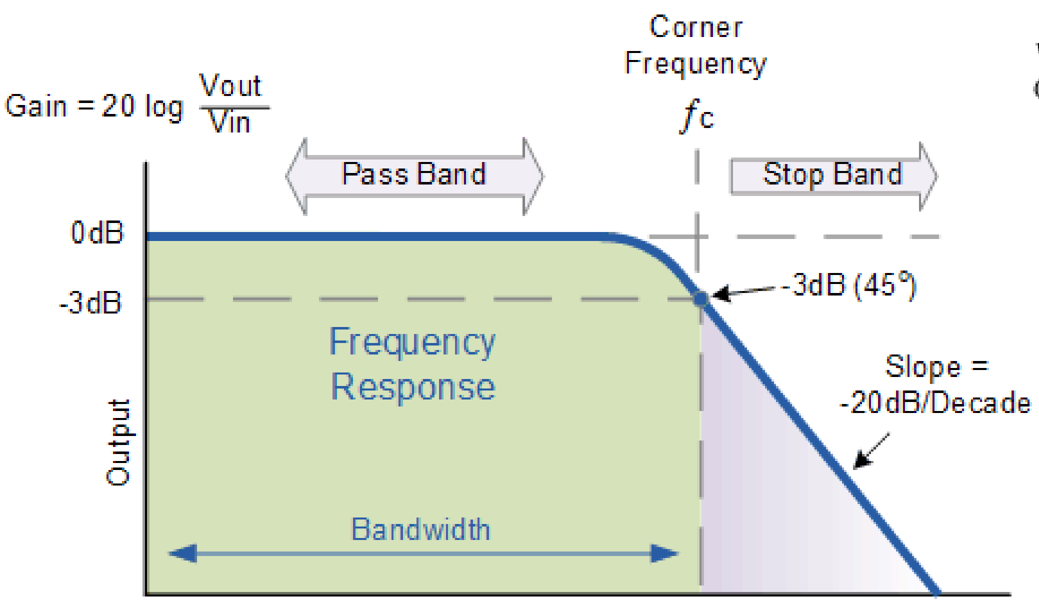

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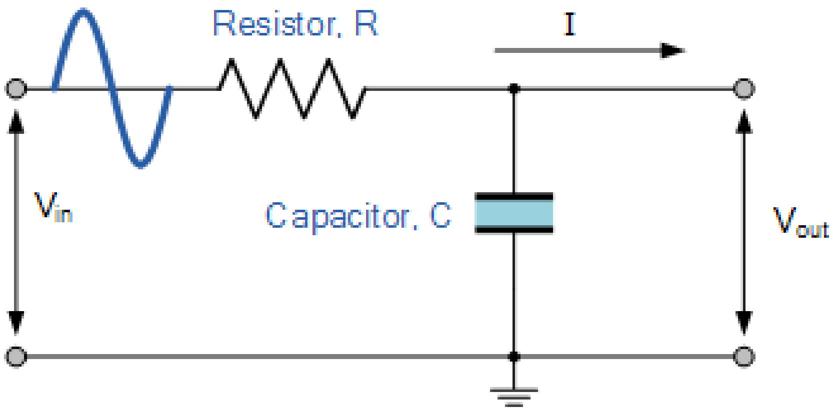

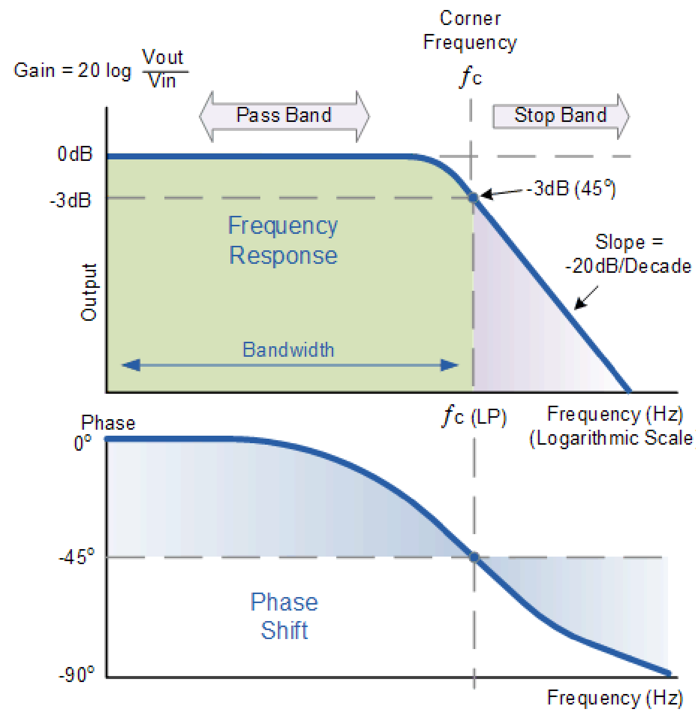

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Dictionary definition of ***ERROR***:

Difference between True Value and Measurement or Calculation

Truth is usually not known – the reason for doing experiments

In scientific analysis, the difference is a ***DISCREPANCY***

What are ERRORS?

1) Illegitimate. Mistake in setup, assumptions, calculations, etc

2) Uncertainties, randomness, statistical fluctuations

3) Systematic

Accuracy *vs* Precision

Accuracy: How close to the truth?

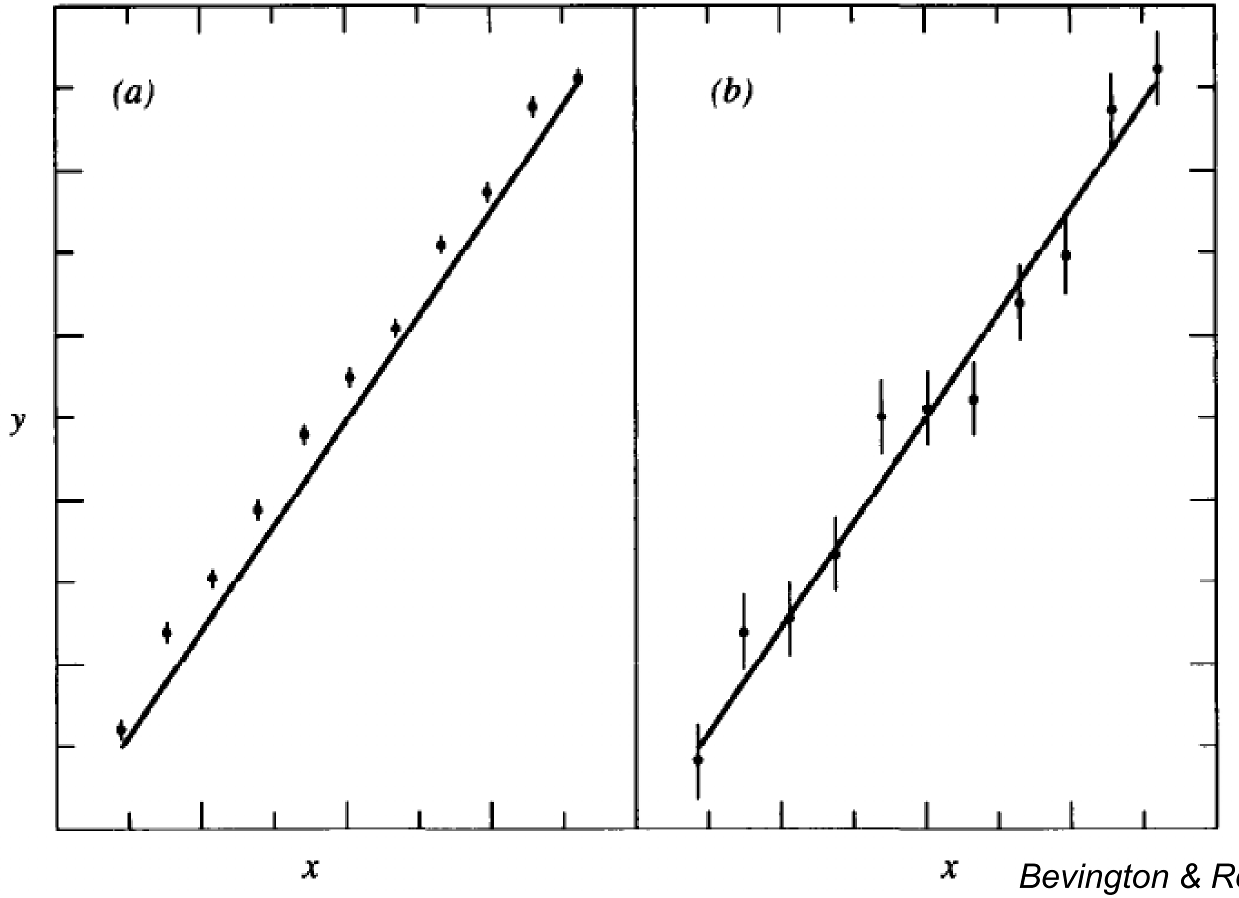
Precision: How well is the result known?

Accuracy = abs(Truth – Measurement)

Precision = Number of significant figures in Measurement

Precision can be high even if **Accuracy** is poor

High precision, inaccurate **Lower precision, more accurate**



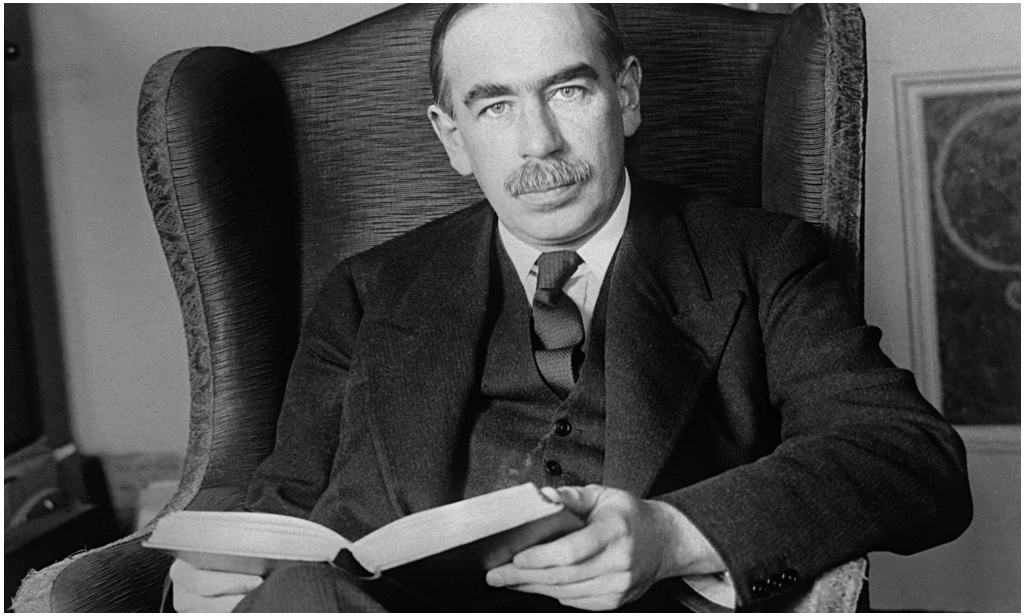
Bevington & Robinson, 3rd ed.

Line: True behavior of $y = f(x)$
Experiment: Data points with error bars

Error bars indicate precision

“It is better to be roughly right than precisely wrong.”

John Maynard Keynes



SYSTEMATIC ERRORS

Systematic Errors harder to identify than random fluctuations

Statistical analysis is usually ineffective

Examples:

- Poor calibration of equipment
- Lack of familiarity with equipment
- Human bias – knowing expected result ahead of time

Avoiding systematic errors: Careful setup, not rushing, experience

RANDOM ERRORS DETERMINE PRECISION

Reduced by improving/refining experimental technique

Better equipment, less noisy

Statistics: Take more data
(although some experiments prevent this)

SIGNIFICANT FIGURES and ERROR BARS

$$1.60217662 \pm 0.1 \times 10^{-19} \text{ coulombs}$$

very precise

not very accurate

Probably should be written this way:

$$1.6 \pm 0.1 \times 10^{-19} \text{ coulombs}$$

ERROR PROPAGATION

$$s_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 s_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 s_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 s_z^2 + \cdots}$$

https://ghz.unm.edu/education/juniorlab_pdfs/taylorformulas.pdf

STATISTICS AND RANDOM ERRORS

Variation between multiple measurements of same quantity

As number of measurements increase, pattern emerges from data

Pattern distributed around the correct value (assuming no systematic error)

Average value of *x* after *N* measurements:

$$\overline{x} = \frac{1}{N} \sum_i x_i$$

What are statistics?

- A statistic summarizes data (data reduction)
- Statistics are the basis for using the data to make a decision
- Example: Is the faint smudge on an image a star or a galaxy?
    - Measure FWHM of the point-spread function.
    - Measure full-width-half-maximum, the FWHM.
    - The data set, the image of the object, is now represented by a *statistic*

What is statistical analysis?

- 1. Formulate a hypothesis
- 2. Gather data to test the hypothesis (via experiment, or by finding existing datasets)
- 3. Compare with the expected probability of that result (the sampling distribution)

Problems:

We don’t know the actual underlying distribution

Small sample size

Important uses of statistics

- Statistics can create precise statements for stating the logic of what we are doing and why
- Statistics allow us to quantify uncertainty
    - Measured quantities are basically useless without some measure of the associated range/error
    - Sometimes this can be inferred, but much better to be explicit (e.g. 5 photons, 72.1 degrees)
- Statistics help us avoid pitfalls like confirmation bias
- Statistics help make decisions about data

Common uses of statistics

- Measuring a quantity (parameter estimation)
  - Given the data, what is the best estimate of a particular parameter? What is the uncertainty in that estimate?
- Searching for correlations
  - Are two variables correlated, and is there an underlying physical mechanism?
- Testing a model
  - Given some data and a model, are the data consistent with the model? Which model best describes the data?

Median value of a data ensemble $m_{1/2}$

Half of all data > $m_{1/2}$

Half of all data < $m_{1/2}$

Deviation of a data point about the **mean**: $d_i = x_i - \overline{x}$

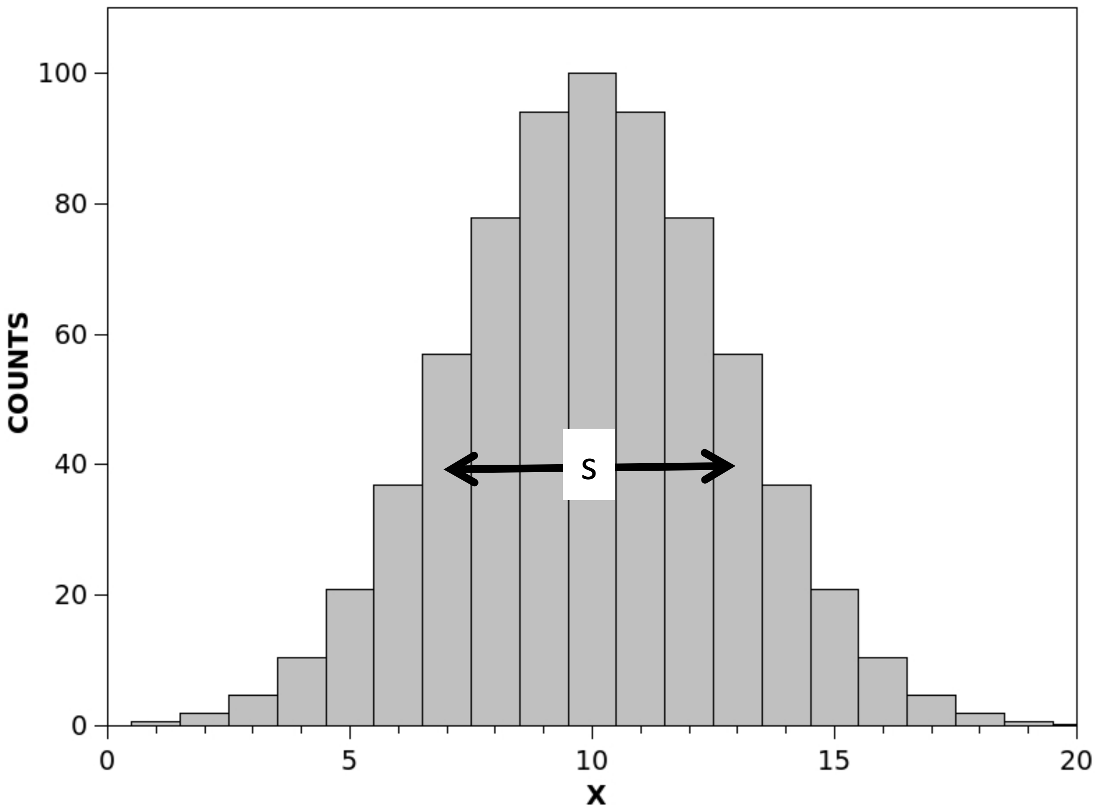
Average deviation: $\overline{d} = \overline{x} - \overline{x} = 0$ *Not useful*

Variance: $$\sigma^2 = \frac{1}{N-1}\sum_i^N d_i^2 = \frac{1}{N-1}\sum_i^N (x_i - \overline{x})^2$$

Standard deviation: $\sqrt{\sigma}$

PROBABILITY DISTRIBUTION

If the value of x is random: **GAUSSIAN** distribution



EXAMPLE

Most probable value: x = 10 (Mean)

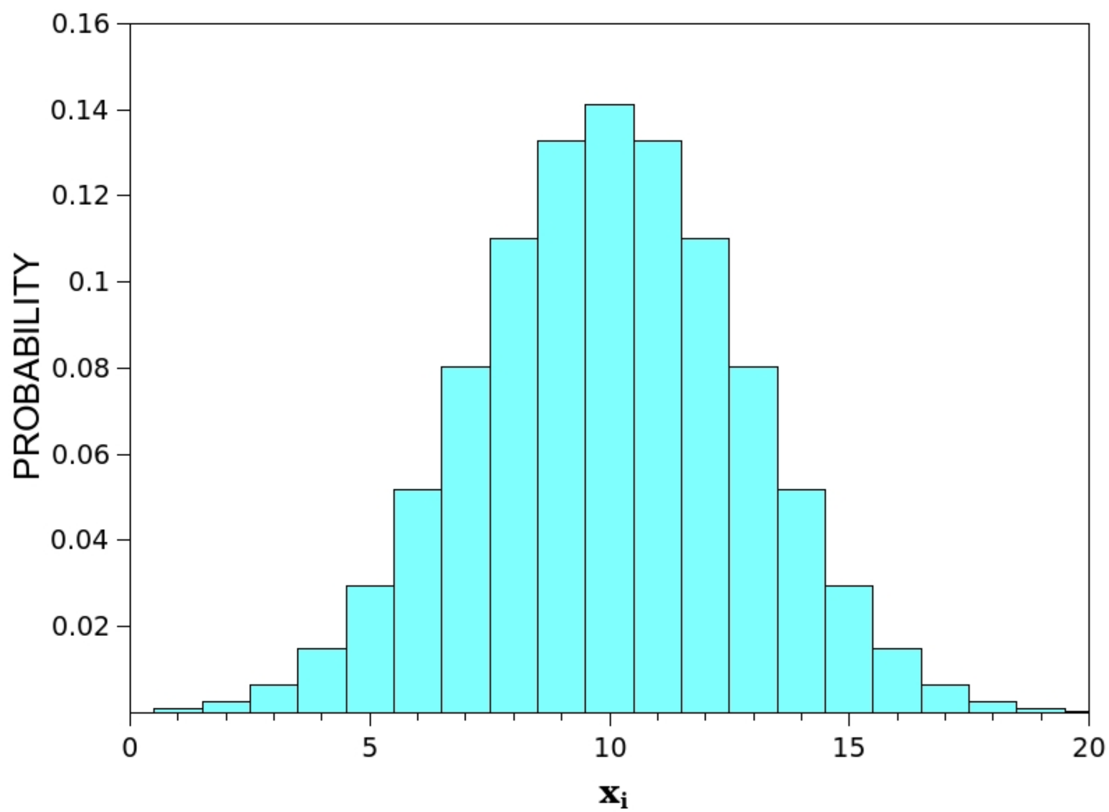
Variance: $s^2 = 8$

Standard deviation: $s = 2.82$

Probability p_i that x will have a specific value x_i

Probabilities must sum to 1: $\sum_i^N p_i = 1$

Expectation value: $\langle x \rangle = \sum_i^N x_i p_i = \overline{x}$



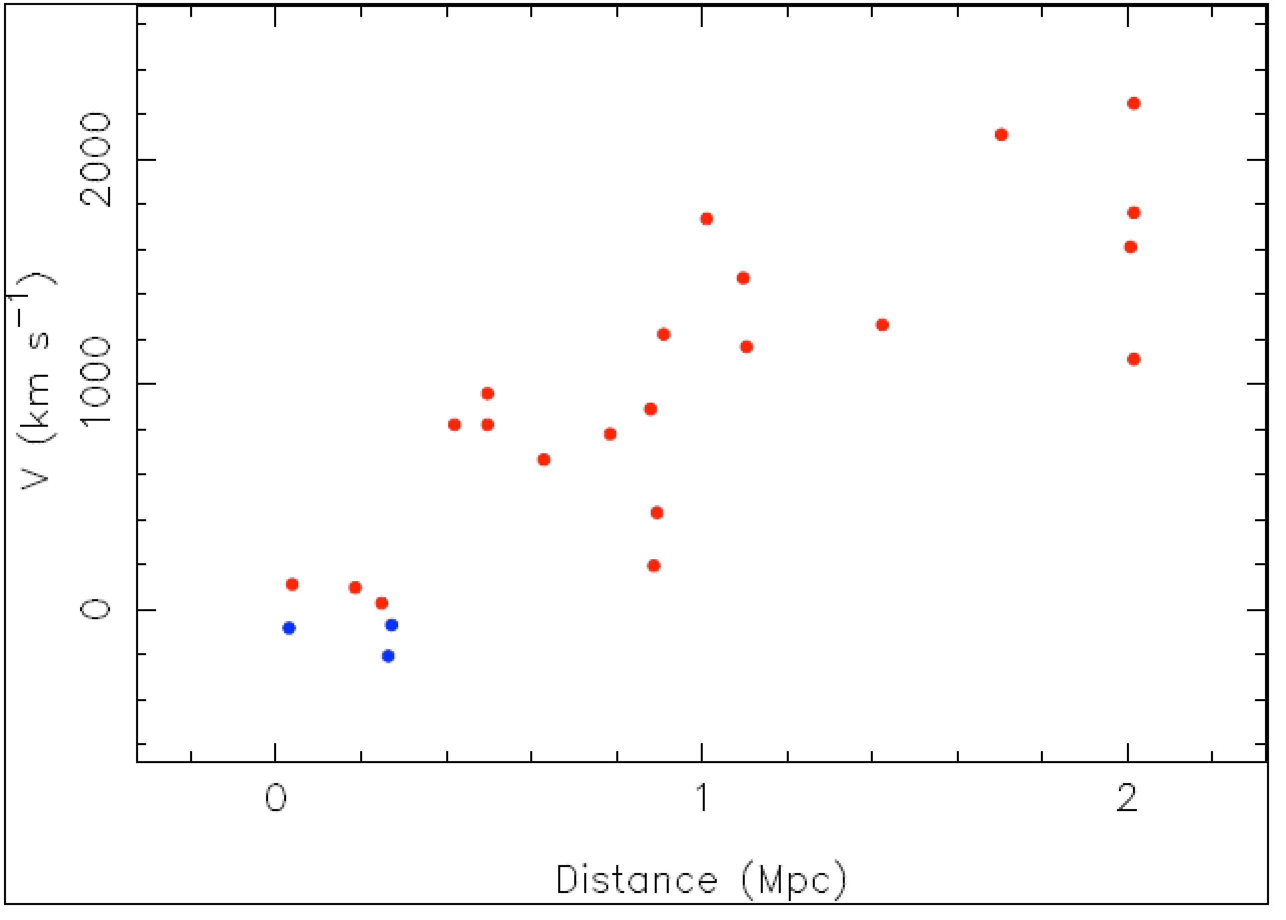
Correlating data

- When looking at new measurements, it is instinct to try to correlate it with other results
  - Checking if our measurements are reasonable
  - Checking if other results are reasonable
  - To test a hypothesis
  - Shot in the dark
- There are a few common traps to fall into when attempting to find correlations

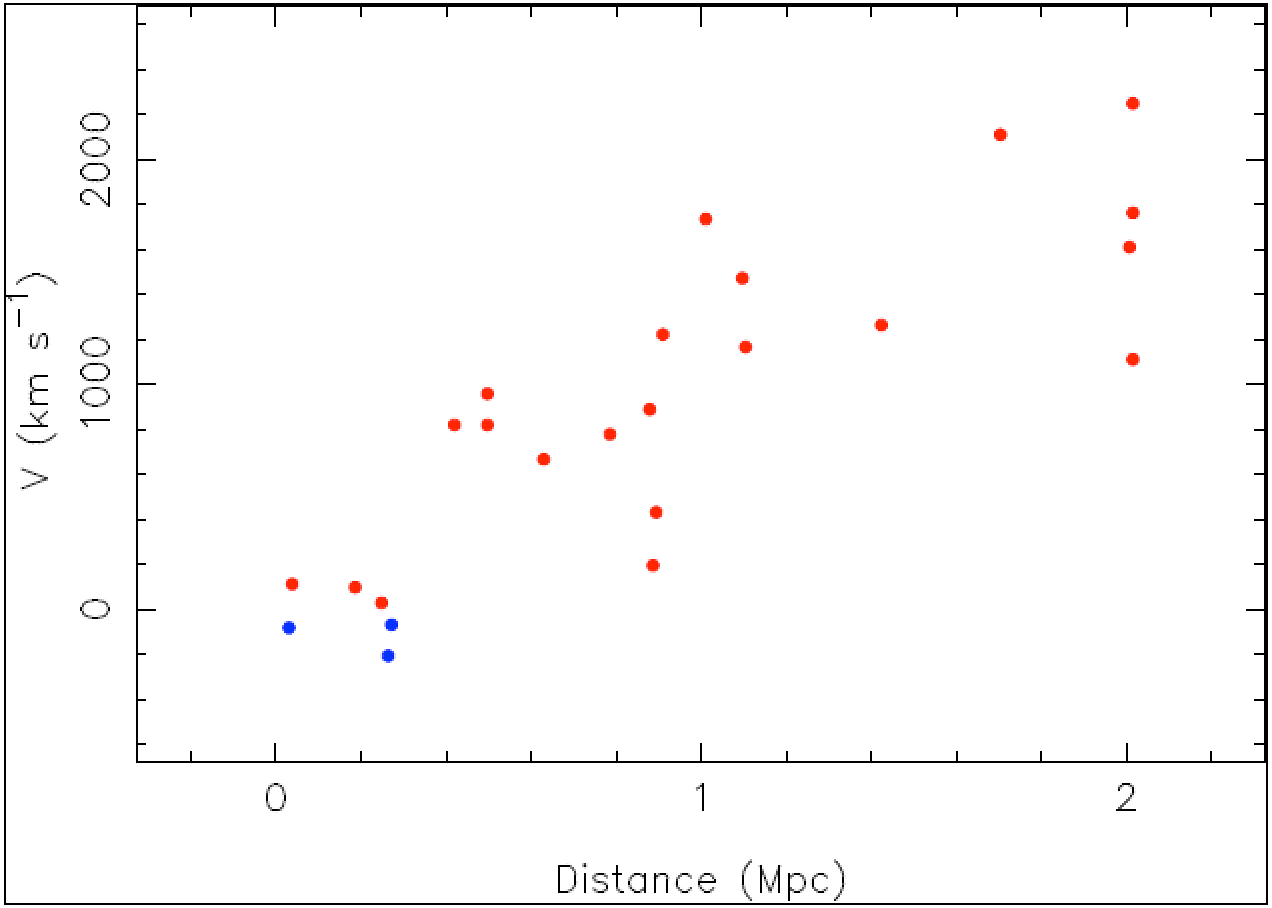
Fishing trips

- Correlation does not prove a causal connection!
  - Examples of correlations
    - Number of violent crimes in cities versus number of churches
    - The quality of student handwriting versus their height
    - Stock market prices and the sunspot cycle
    - Cigarette smoking vs lung cancer
    - Health vs alcohol intake
- Potential reasons
  - **Lurking third variables**
  - Similar time scales
  - Causal connection

Correlation?



Correlation?

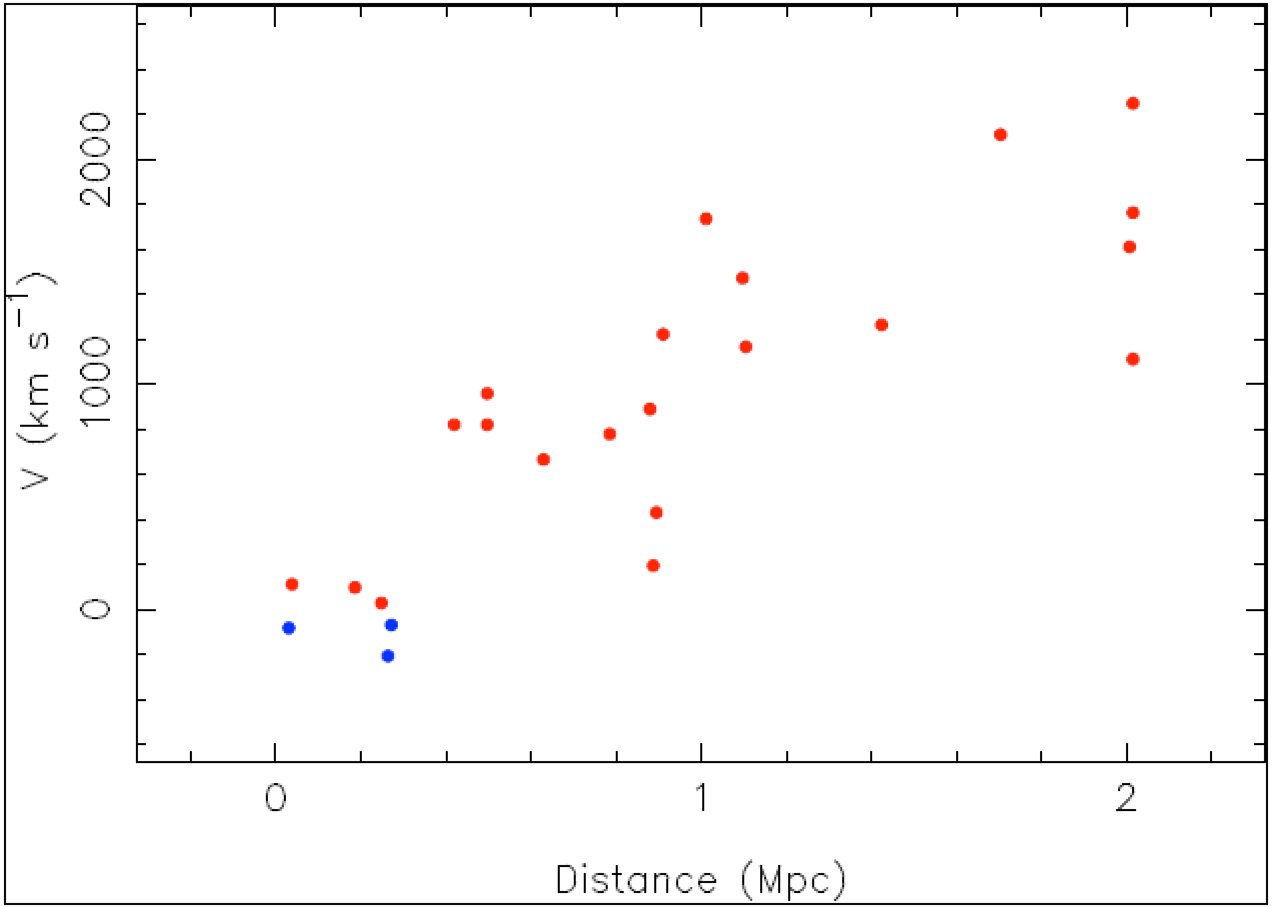


An early Hubble diagram, N=24 galaxies (1936)

So you think your data is correlated

- Time to fit the data to a model!
- “All models are wrong, but some models are useful.”
    - George E. P. Box

Fitting data to a model



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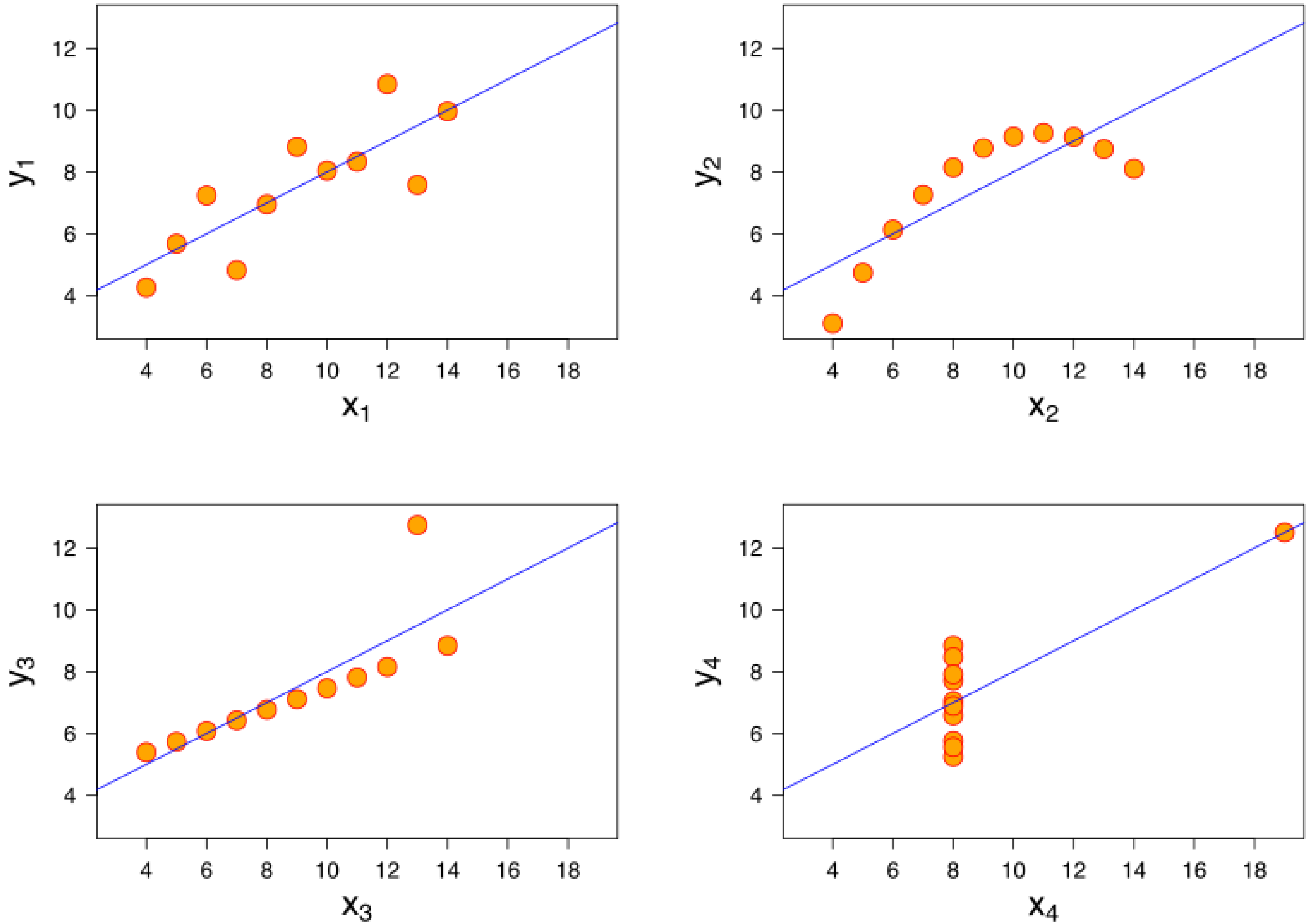
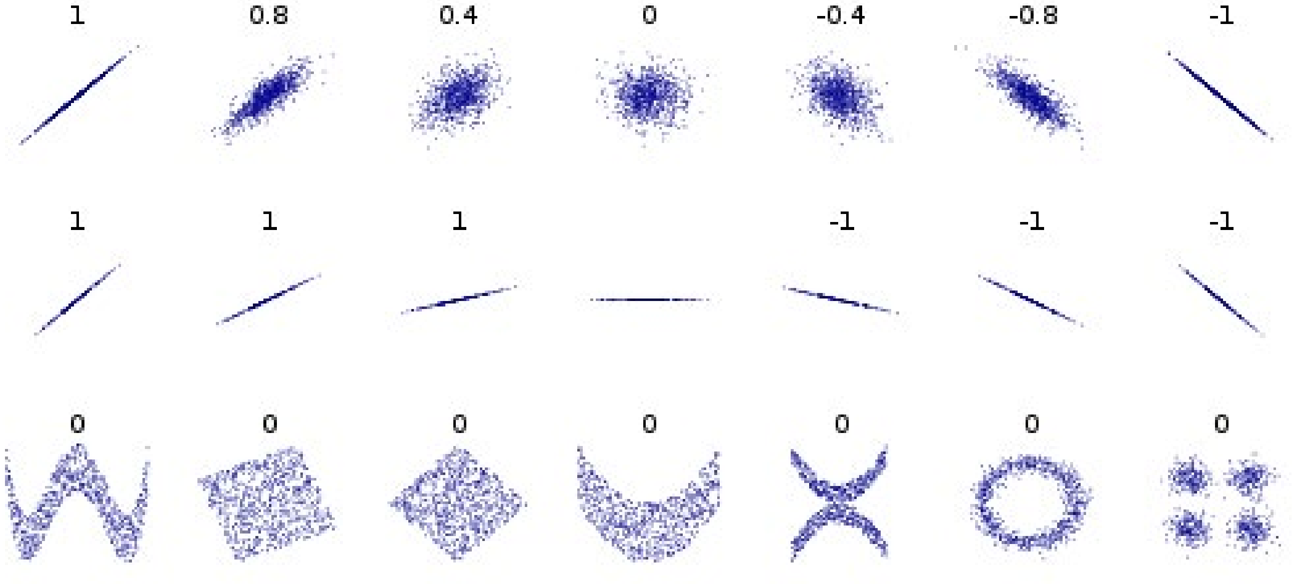


Simplifying correlations

- Linear correlation
  - $y=mx+b$
  - Multidimensional: $z = mx + ny + b$
- Linear correlations are easy to plot and examine
- Can linearize your data to make it a linear correlation
  - Example: Surface brightness of a disk
    - $I(r) = I_0 e^{-r/h}$
    - Linearized form: $\ln(I) = \ln(I_0) - r/h$
  - Also straightforward to do for power laws

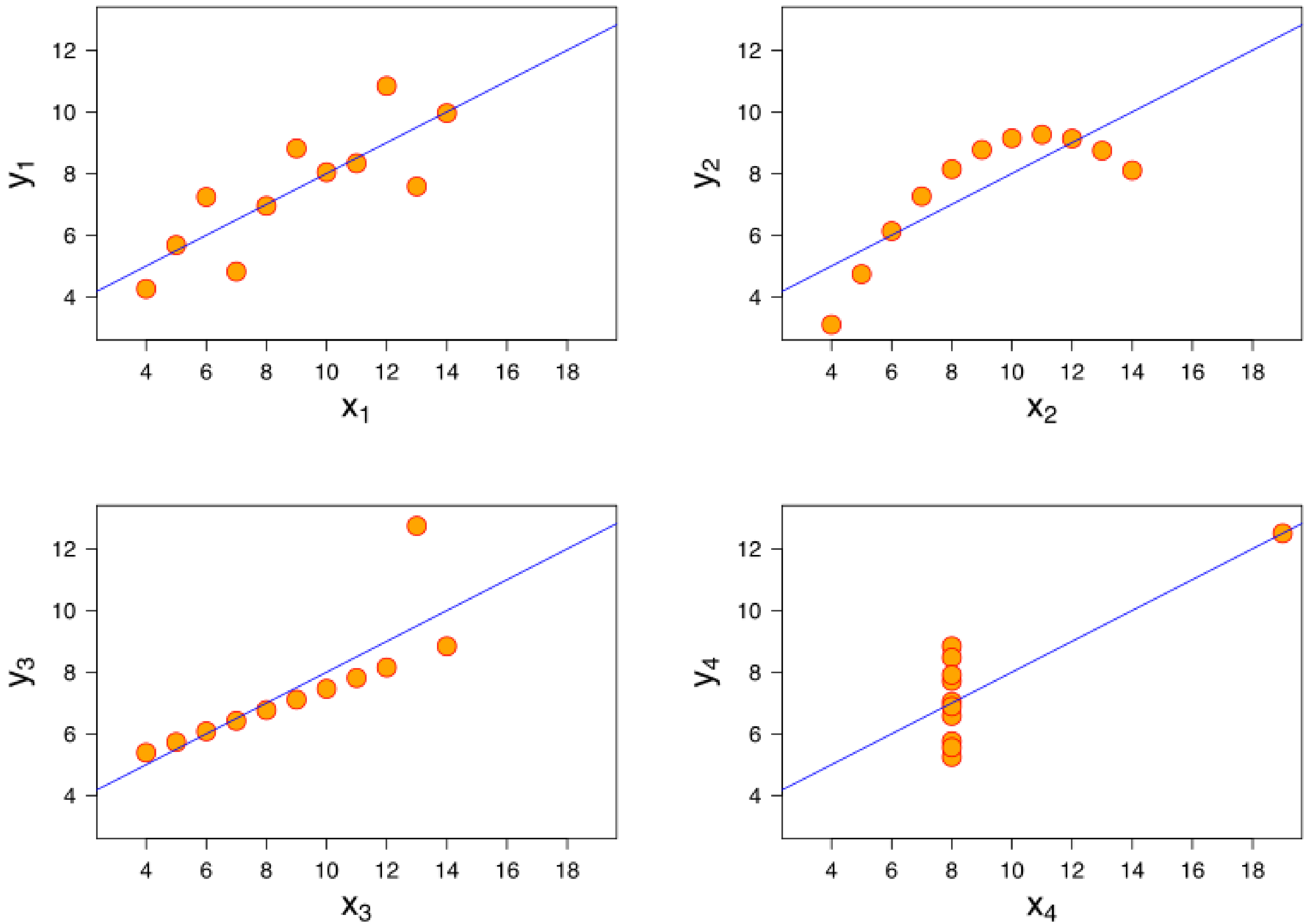
Pearson's correlation coefficient

https://en.wikipedia.org/wiki/Pearson_correlation_coefficient

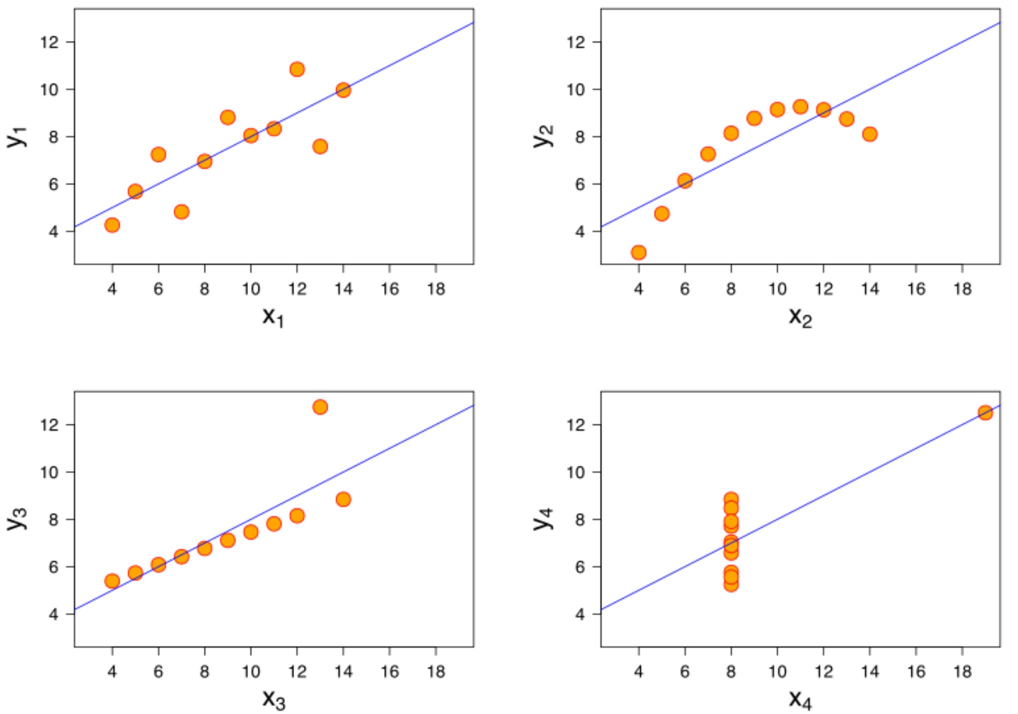


- Pearson's correlation coefficient, r, measures the linear correlation between two variables

Anscombe’s quartet



Anscombe’s quartet



Property	Value	Accuracy
Mean of *x*	9	exact
Sample variance of *x*	11	exact
Mean of *y*	7.50	to 2 decimal places
Sample variance of *y*	4.125	±0.003
Correlation between *x* and *y*	0.816	to 3 decimal places
Linear regression line	$y = 3.00 + 0.500x$	to 2 and 3 decimal places, respectively
Coefficient of determination of the linear regression	0.67	to 2 decimal places

https://en.wikipedia.org/wiki/Anscombe%27s_quartet

Modeling uncertainty

- Least squares fitting
  - A straightforward output of Python/Matlab/Excel/etc
  - Assumes uncorrelated Gaussian statistics
  - Can get different results depending on the exact algorithm, especially for data with a small number of samples, or data with outliers
- Other ways to check uncertainty
  - Jack-knife
    - Go through data and toss out data points, and recalculate
    - Common to split data in half (e.g. first-half vs second-half)
  - Bootstrap
    - Go through dataset picking N points at random, recalculate and look at variation