Chua’s circuit and chaos

Chaos can be found across a wide variety of physical systems. In this lab, we will learn about chaos and simulate chaotic systems, and then we will explore it in a circuit known as a Chua circuit, which demonstrates a Lorenz attractor.

Components of this activity

1. Take notes throughout this activity
    a. Your notes should be a record of the research and activities you did in class, including software used, websites visited, readings that you looked at, and any search terms used when looking for more information. Include screenshots and plots where appropriate. The purpose is to help us follow along with the work that you did in class, and to make it possible for someone to be able to re-create this later.
2. Answering questions
    a. Answer the questions bolded throughout this document
3. Following the activities described below, including
    a. Readings (see folder on Teams Class Materials -> Chaos_Readings for full set of available reading materials)
    b. Simulate the logistics curve using Chaos 101 simulator
    c. Learn about Chua’s circuit and why it exhibits chaos
    d. Build Chua’s circuit at multisim.com -> requires making an account!
    e. Probe the simulated circuit and study its behavior
    f. Simulate the circuit behavior using Matlab

These activities should be completed across 2 full lab sessions. Section (f) may be skipped if there is not enough time.

Section 1 – Exploring Chaos

First, read Chapter 1 of Exploring Chaos by Brian Davies for some background on chaos: https://ghz.unm.edu/education/spring2020/exploringchaos_davies.pdf

The first section discusses **population models** and the logistics model.

Q1. What is the equation for the simple logistics model? What is the meaning of each variable?

The text uses CHAOS FOR JAVA to simulate the logistics curve. This website has an equivalent simulator:

https://insightmaker.com/insight/43643/Chaos-101-Logistic-Equation

The website allows you to simulate two curves at once (X1, R1 for one simulation, X2, R2 for a second simulation), and see the difference between two curves for slight differences in initial conditions.
You can use that simulator, or you can make your own simple simulation in Python or Matlab!

Q2. Reproduce the 4 plots shown in Figure 1.1, and take a screenshot of each one.

Play around with the simulations, including trying out the values described but not shown in the text.

Q3. Using the Chaos 101 simulator, demonstrate chaotic behavior with the logistics equation. Describe the parameters chosen, and why this demonstrates chaos. Include screenshots AND a description of what you simulated.

Section 2 – What is Chaos?

Pages 1 and 2 of this pdf describe the characteristics of chaos:

https://ghz.unm.edu/education/spring2020/chaostheorytamed_williams.pdf

Q4. List 3 of these characteristics which are requirements for a system to be chaotic.

Q5. List 3 of these characteristics that are a consequence of a system being chaotic.

For the next question, you can use the reading in the two pdfs provided, or search for other examples online, but use the criteria from "Characteristics of Chaos" as the requirements and consequences of chaos.

Q6. List 2 examples of chaotic systems, and give one requirement that they meet which results in them being chaotic, and one important consequence of their chaotic behavior.

Section 3 – Learning about Chua's Circuit

Q7. Chaotic behavior requires having some nonlinear element. Let's do some circuits 101 refreshers before we dig into the details of Chua's circuit

a. Sketch an example plot of voltage (x-axis) vs current (y-axis) for a resistor with a voltage applied to it. What equation does it follow? What kind of behavior does it exhibit?
b. Read this page to learn about what a diode is and how it behaves: https://learn.sparkfun.com/tutorials/diodes/real-diode-characteristics

   Sketch an example of voltage vs current for a **diode**. What is special about its behavior?
c. Can you sketch a simple voltage vs current plot for a capacitor or an inductor? Why not?

Q8. Read this article on the basics of Chua's circuit and chaos, and give a brief summary of what Chua's circuit is, and why it is special : http://www.scholarpedia.org/article/Chua_circuit

Q9. What is a Chua Diode? Why is it called a diode? Sketch the voltage vs current plot for a Chua Diode.

Section 4 – Building Chua's circuit

We will use multisim.com to construct Chua's circuit. There are example circuits that you can copy. You can use these as a reference, but you should build the circuit yourself from scratch. This will help you understand the components, and how to use Multisim (this is very similar to many widely used circuit simulation tools).

You will need to make an account on Multisim. This should be straightforward.

Once you have made an account, let me know so I can add you to our group so I can view your circuit, and you can see the version of Chua's circuit I have added.

First, make a practice circuit to get used to using Multisim. Start a new circuit, making a simple circuit with a resistor and a voltage source. Use probes to measure both the voltage and current. The "grapher" tab is like an oscilloscope, showing the signal vs time.

Note: I couldn't figure out how to plot voltage vs current (instead of vs time), possibly because this is free software. You can try for a few minutes to see if you can get it to plot that, but I think you have to download the data and do it in another program.

Change the voltage to a few different values, to see how the voltage and current change in the grapher view.

At the upper right, there is an arrow icon to export data. Export the grapher data as a .csv file, and use Python, Excel, or similar, to load this data and plot voltage vs current. The voltage was controlled manually, so that should be the x-axis (independent variable). **Be sure to include this plot in your lab notebook along with a description of how you generated it.**

Change the resistor to a diode, and repeat the measurement, downloading the data and plotting voltage vs current, and take screenshots and include plots in your lab notebook.

Share this circuit with the group so that I can see it (ask if you can't find the share feature).

Next, we'll look at Chua's circuit using the op-amp-based "gyrator."

Read the pdf called "BuildingChuasCircuit.pdf" – this describes how to build the circuit.

There is one special component that you will need, the LMC6482.
Go to the group "Physics 307L - Simulating Chaos with Chua's Circuit" and you should see a shared circuit called "Chua's Circuit – Starter."
Copy this to your account so that you can modify it and build the circuit around the LMC6482.
You should be able to build a Multisim circuit that is identical to the diagram in
the pdf describing the circuit.

Note: There is also a completed Chua's circuit in the same group folder, which you can compare to the *one that you build.*

Use the probes to measure the voltages across the two capacitors.
Display the voltage across C_1 on channel 1, and then across C_2 on channel 2.
The behavior we want to see is in plotting these two voltages against each other (like we plotted voltage vs current in the previous example. If we had built this in lab, we would change the oscilloscope display mode to XY.
Instead, we can download the time-series data and make plots in another program (like we did for the simple circuit).

Vary the resistance, *R* (the top resistor in the circuit), and make "XY" plots of the two voltages. **Make sure you understand what you're seeing before you proceed.**

Describe what you are seeing, for each value you try for *R*. Many behaviors can be observed including dc equilibrium, period doubling, single Rossler-type attractor, strange attractor (double scroll), large limit cycle beyond the double scroll.

Q10. By varying the resistance, determine the minimum (R_{min}) and maximum (R_{max}) values of the variable resistance that give chaotic behavior (*i.e.*, two distinct attractors should be evident).

Q11. Now even more fun: measure the *relative time* that the voltages spend in each attractor as a function of R_v. Select five resistance values evenly spaced between R_{min} and R_{max}, and try them out for resistor *R*.
Export the data to CSV for each value, and use another program to calculate the relative time spent in each attractor.
Plot the relative time that the oscillator spends in the positive-voltage basin vs R_v for the data that you collected.

Section 5 – Simulating Chua's Circuit

Download the MATLAB simulation of Chua's circuit from chuacircuits.com.
http://www.chuacircuits.com/matlabsim.php

Edit the code so that it uses the component values that you used in Section 4 in Multisim

Run the simulation for each of the R_v values from Section 4 and (qualitatively) compare the simulation to the data that you collected. Also see if you can re-create some of the other observed o-scope patterns!