Lab 9: Chaotic Circuit

Adapted from:

Chua's Circuit for High School Students
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Introduction:

Have you ever noticed water dripping from a tap, which is sometimes regular and sometimes irregular? Or water flowing through an obstacle in a way that can be either smooth (laminar) or turbulent?

At some point in time we all notice these phenomenon, however, they are usually very hard to explain. It was only after Edward Lorenz [Lorenz, 1963] came to conclude that his computer simulated weather model was highly sensitive to initial conditions that the scientific community rigorously studied the phenomenon of chaos. It took more than 30 years to prove that the observations by Lorenz were indeed chaotic [Tucker, 2000].

Chaos, along with Quantum Mechanics and Relativity, has been hailed as one of the major discoveries of the 20th century. Chua's circuit was the first circuit implementation specially designed to exhibit chaos [Chua, 1984], and the first that was proven to be chaotic rigorously [Chua, et. al., 1986]. Chua's circuit is also the simplest [Chua et. al., 1986] physical system where chaos can be observed.

[Lorenz, 1963] "Deterministic Nonperiodic Flow," E.N. Lorenz, *J. Atmos. Sciences* **20**, 130 – 141 (1963).

[Tucker, 2000] "The Lorenz Attractor Exists," W. Tucker, *Nature* **406**, No. 6799, 948 – 949 (2000).

[Chua, 1984] "The Genesis of Chua's Circuit," L.O. Chua, *Archiv fur Elektronik und Uebertragungstechnik* **46**, No. 4, 250 – 257 (1992).

[Chua, et. al., 1986] "The double scroll family, Parts I and II," L.O. Chua, T. Matsumoto, and M. Komuro, *IEEE Trans. On Circuits and Systems* **CAS-33**, No. 11, 1072 – 1200 (1986).

[Kennedy, 1992] "Robust Op-Amp Realization of Chua's Circuit," M.P. Kennedy, *Freqz.* **46**, No. 3-4, 66-80 (1992).

Introduction to Chua's Circuit:

A schematic of Chua's circuit is shown in Figure 1. The circuit consists of three energy storing elements (two capacitors C_1 and C_2 and one inductor L), one linear resistor R, and one non-linear "resistor" described by the current versus voltage characteristics ($i_R = f(V_R)$) as shown in Figure 2.

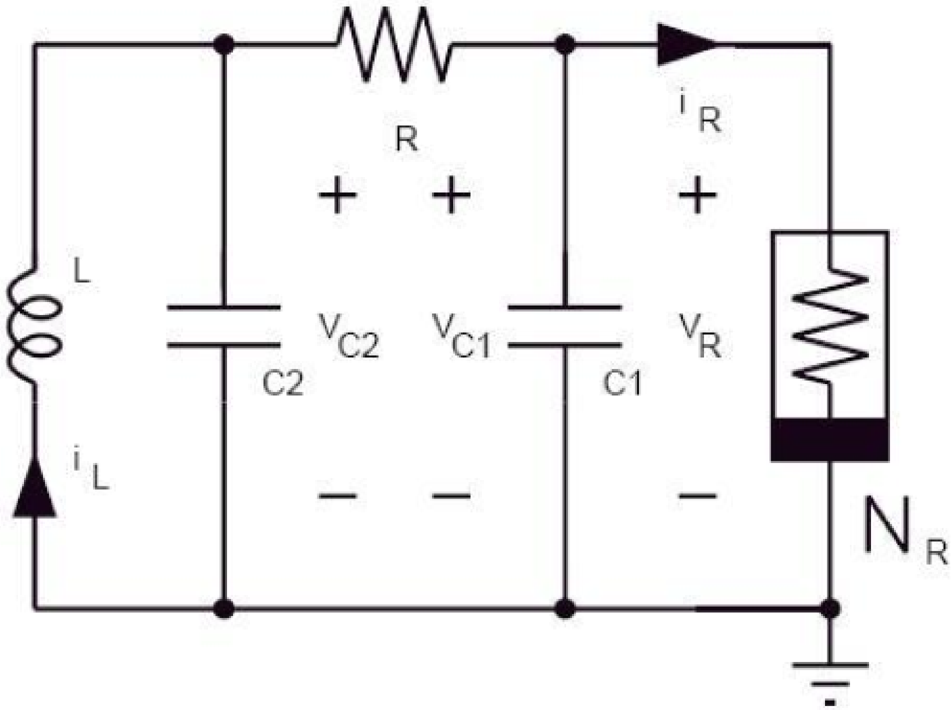

Figure 1. Chua's Circuit

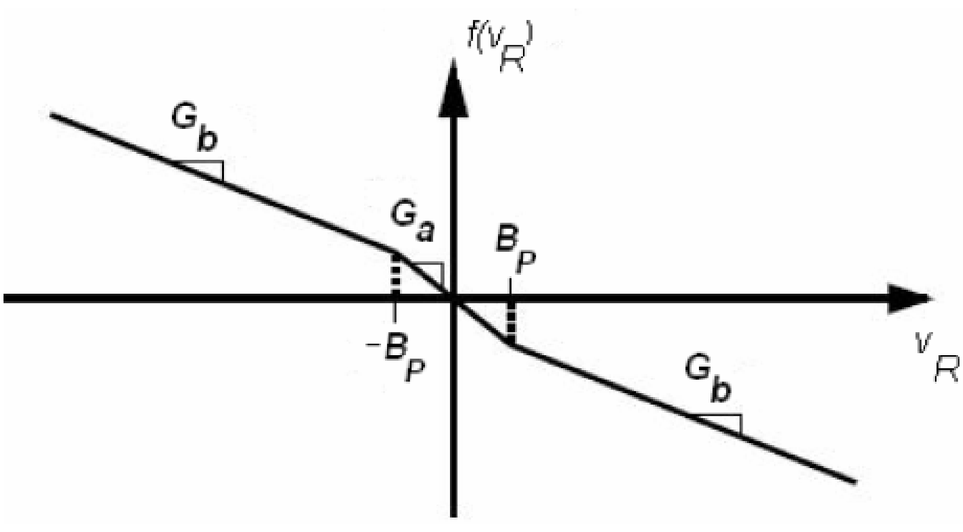

Figure 2. Current vs. voltage characteristics of the Chua Diode

Building the Circuit:

Start by building the nonlinear element of the Chua circuit which is known as the Chua diode (Figure 3). We'll be using a two op-amp implementation of the Chua diode from Kennedy [Kennedy, 1992]. The op-amp that we will be using is the LMC6482. The pin-out diagram is shown below in Figure 4.

As you work on building the Chua diode, I recommend holding off on connecting the 9V batteries. We can connect those at the very end when the rest of the circuit has been built! This will save battery power!

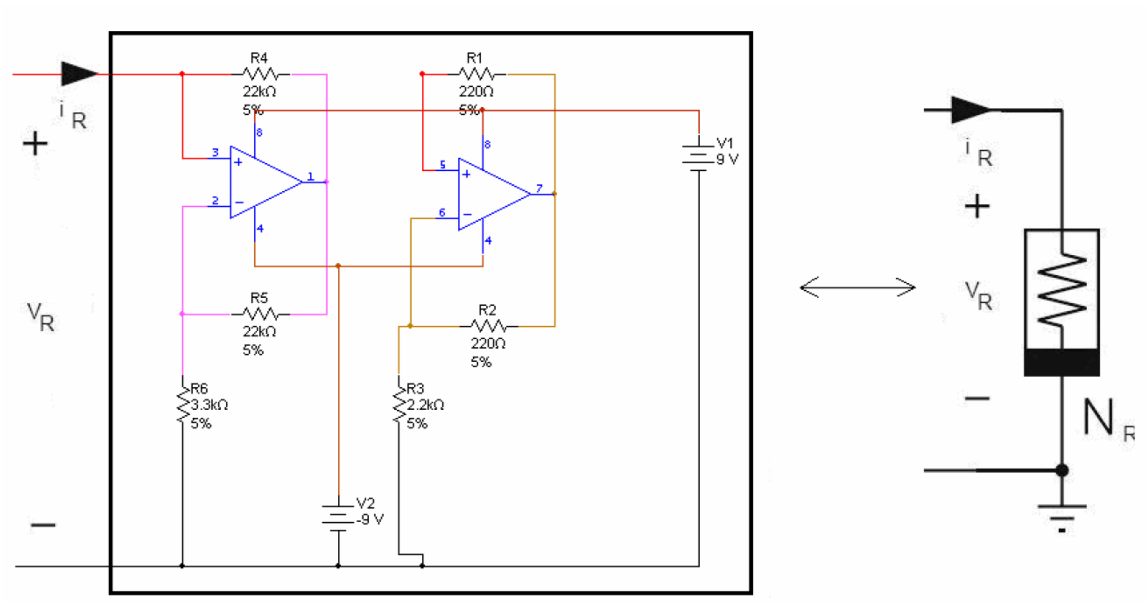

Figure 3. Chua diode realized using M.P. Kennedy's two op-amp realization

Note that R1 = R2 = 220 Ω

R3 = 2.2 kΩ

R4 = R5 = 20 kΩ

R6 = 3.3 kΩ

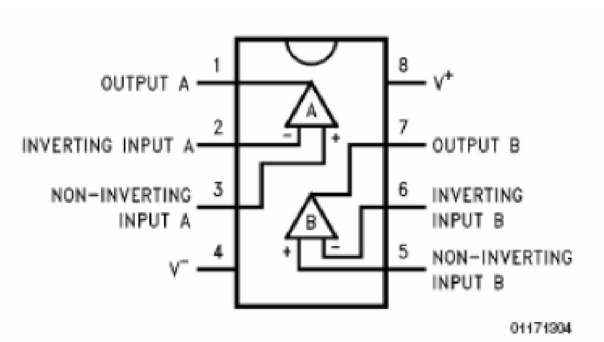

Figure 4. LMC6482 op-amp pinout

Once you have the Chua diode built (sans connecting the batteries) you can finish building the circuit. Connect the two capacitors, a 2 kΩ potentiometer, and the inductor as shown in Figure 5. Also note that you need to connect pin 5 to pin 3 on the op-amp!

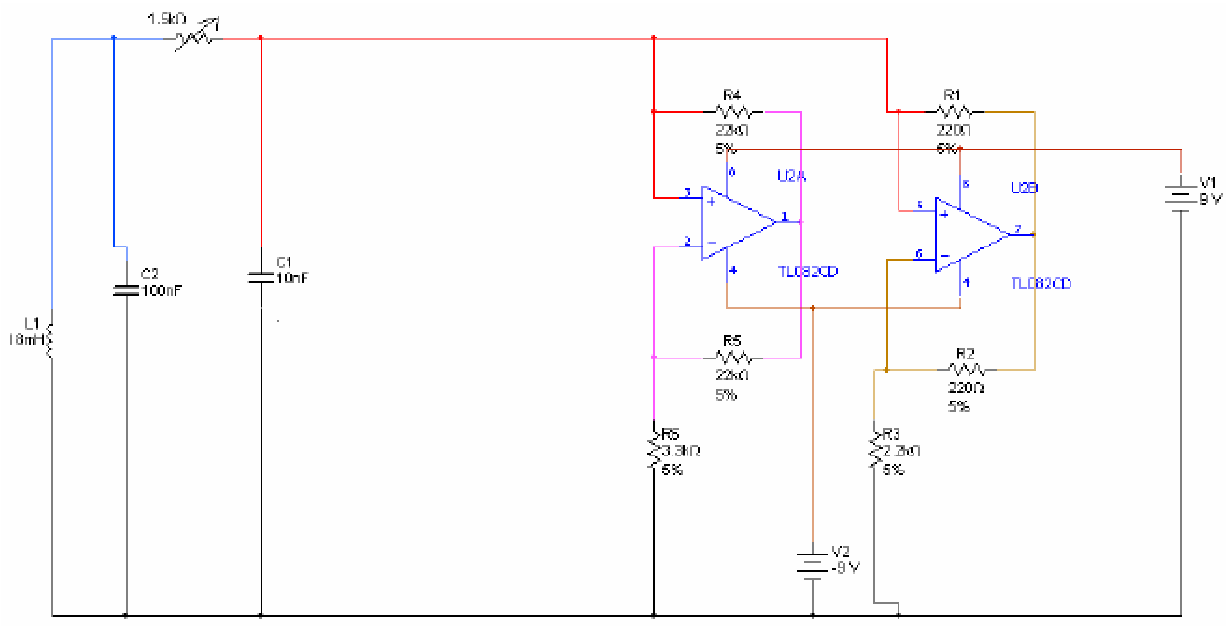

Figure 5. Chua's circuit

Before you try your circuit there are two important remaining steps:

1) Connect the red lead of one of your batteries to pin 8. Connect the black lead of your other battery to pin 4.
2) You need to connect all of your ground points! The ground side of your inductor and the two capacitors needs to connect to the ground side of R3 and R5. In addition, the **black lead of your +9V** supply and the **red lead of your -9V** supply also need to tie into the same common ground point. You may have initially established these ground points on more than one bus, so you need to connect the bus lines. (Remember, the op-amp isn't directly connected to ground, but it still needs to know where ground is, hence the need for a single ground potential.)

Now you are ready to see the chaos! Use your 10X probes: display the voltage across the 10 nF capacitor on CH1 and the voltage across the 100 nF capacitor on CH2. (Make sure your alligator clips are connected to the ground point.) Put the o-scope in XY mode. Use the 2 kΩ potentiometer to slowly vary the signals and eventually find the double scroll pattern (Lorenz attractor). Continue to adjust the pot and see what other patterns you can find!

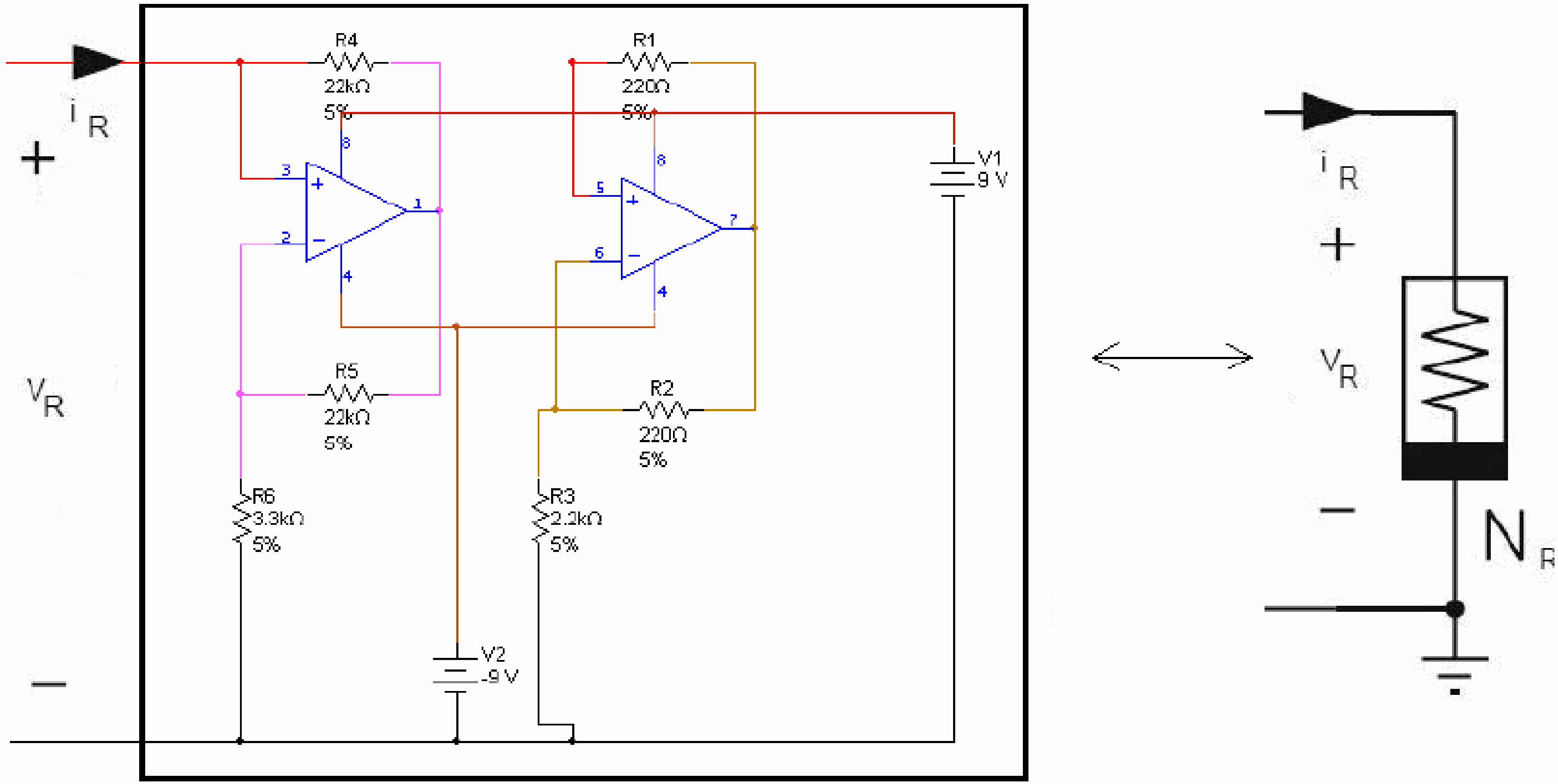

Figure 3. Chua diode realized using M.P. Kennedy's two op-amp realization

Note that R1 = R2 = 220 Ω

R3 = 2.2 kΩ

R4 = R5 = 20 kΩ (not 22 as shown in figure)

R6 = 3.3 kΩ

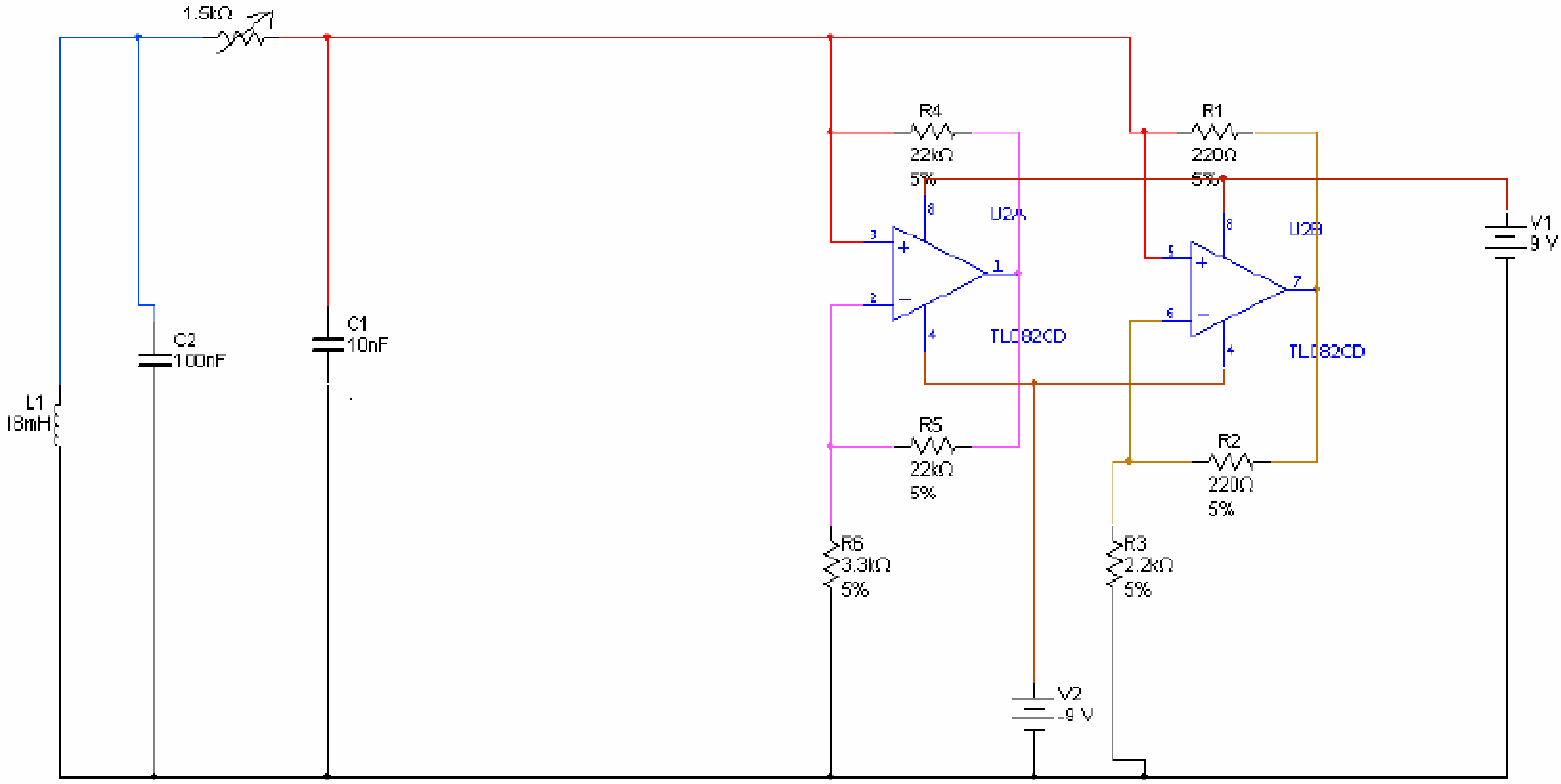

Figure 5. Chua's circuit