Astro 426/526

Fall 2019

Prof. Darcy Barron

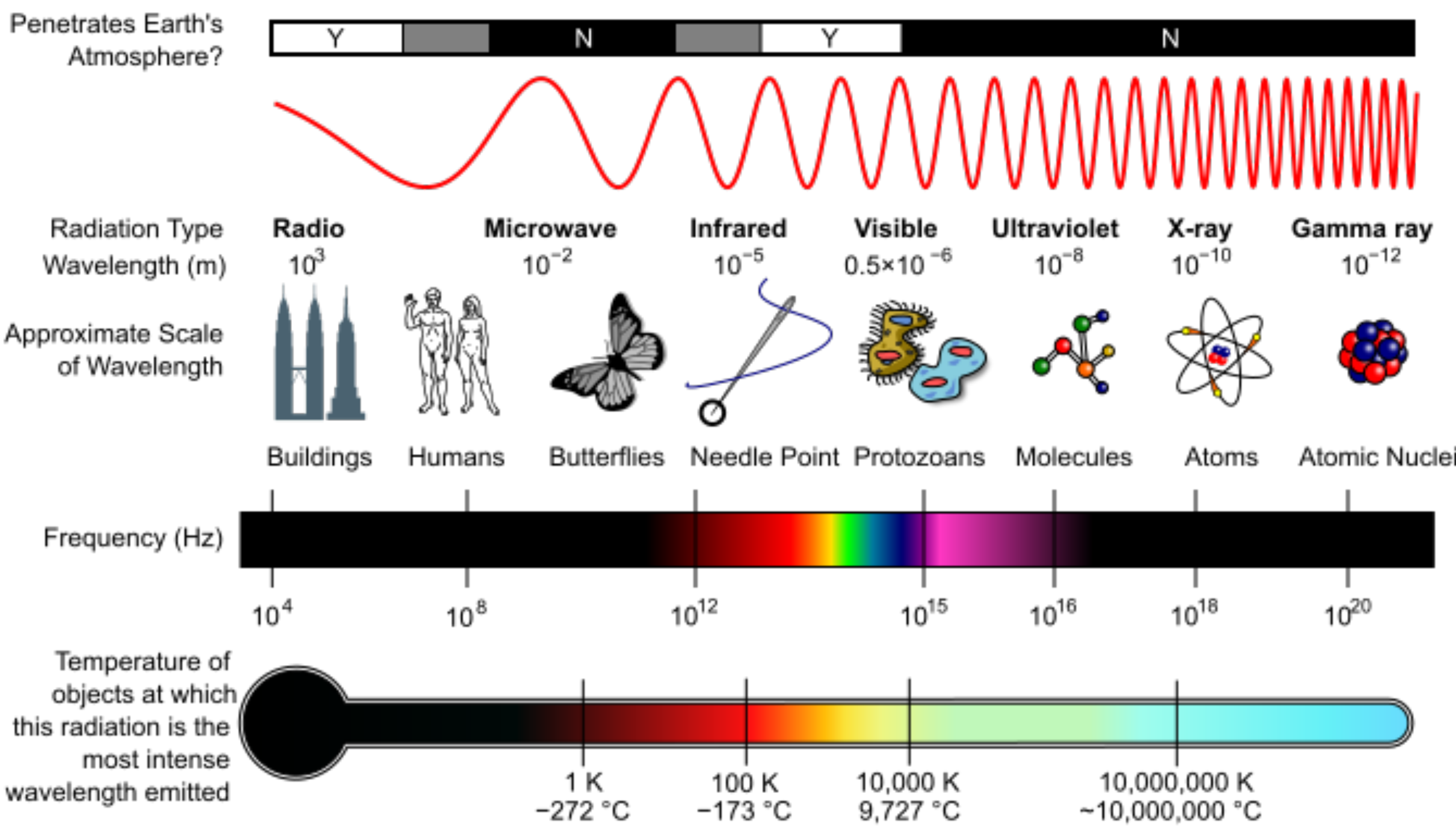
Lecture 12: Review

Midterm on Wednesday

- Open book, both textbooks and calculator allowed
  - One page of notes allowed (also ok to have marked-up textbook)
  - Not allowed: multiple pages of notes, computers, phones, internet
- We will hand out exams quickly to give you the full 75 minutes

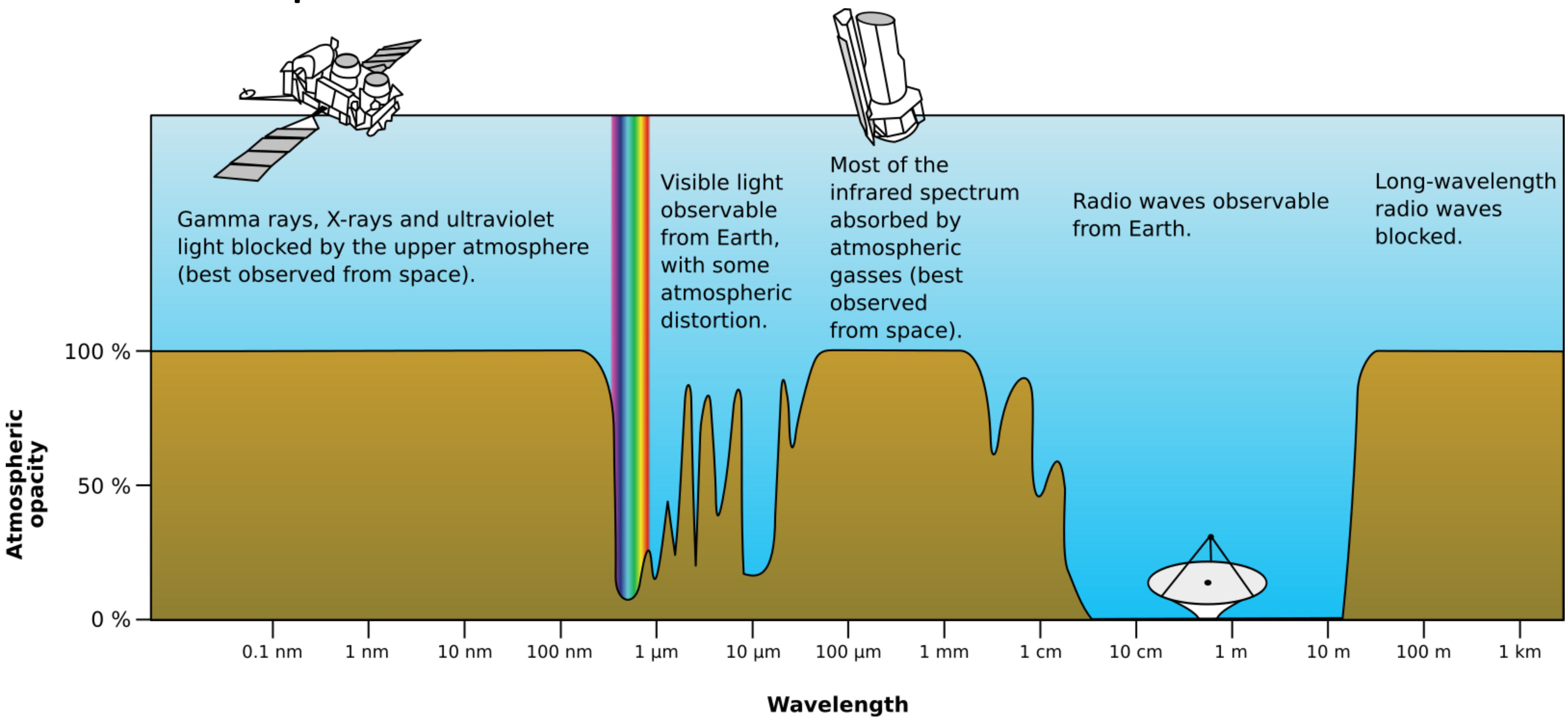
Midterm topics

- Chapters 1 – 3 of Measuring the Universe
    - Chapter 1: **Radiometry, optics, statistics**
    - Chapter 2: **Telescopes**
    - Chapter 3: **Detectors for the ultraviolet through the infrared**
- Chapter 3 of *Practical Statistics for Astronomers*: **Statistics and Expectations** (which heavily references Section 2.4: Probability distributions)
- Background material necessary to understand those chapters of textbook (in lecture slides and notes)

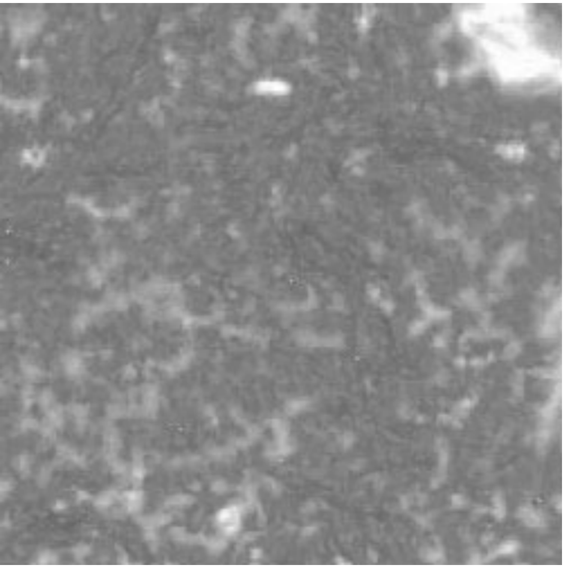


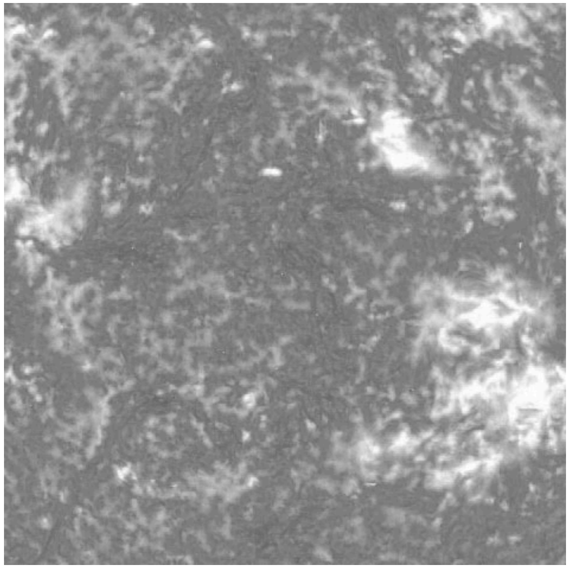
Penetrates Earth's Atmosphere?
Y
N
Y
N
Radiation Type
Wavelength (m)
Radio
10^3
Microwave
10^{-2}
Infrared
10^{-5}
Visible
0.5×10^{-6}
Ultraviolet
10^{-8}
X-ray
10^{-10}
Gamma ray
10^{-12}
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^4
10^8
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of objects at which this radiation is the most intense wavelength emitted
1 K
−272 °C
100 K
−173 °C
10,000 K
9,727 °C
10,000,000 K
~10,000,000 °C

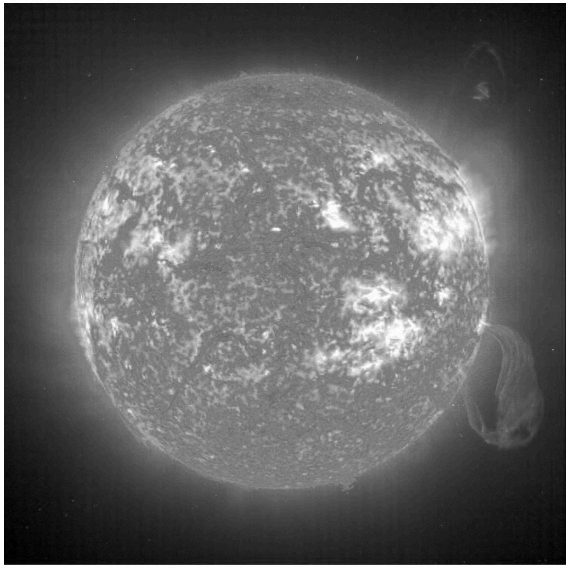
Electromagnetic spectrum and our atmosphere



Brightness







Number of photons falling on the detector *per unit area per unit time per unit solid angle* does not change. This is called **brightness** or **intensity**
The solid angle that the detector sees stays the same.
The solid angle that the sun subtends does change with distance (or aperture size), and so does the total amount of flux received.
(Only true if there is no loss in the system)

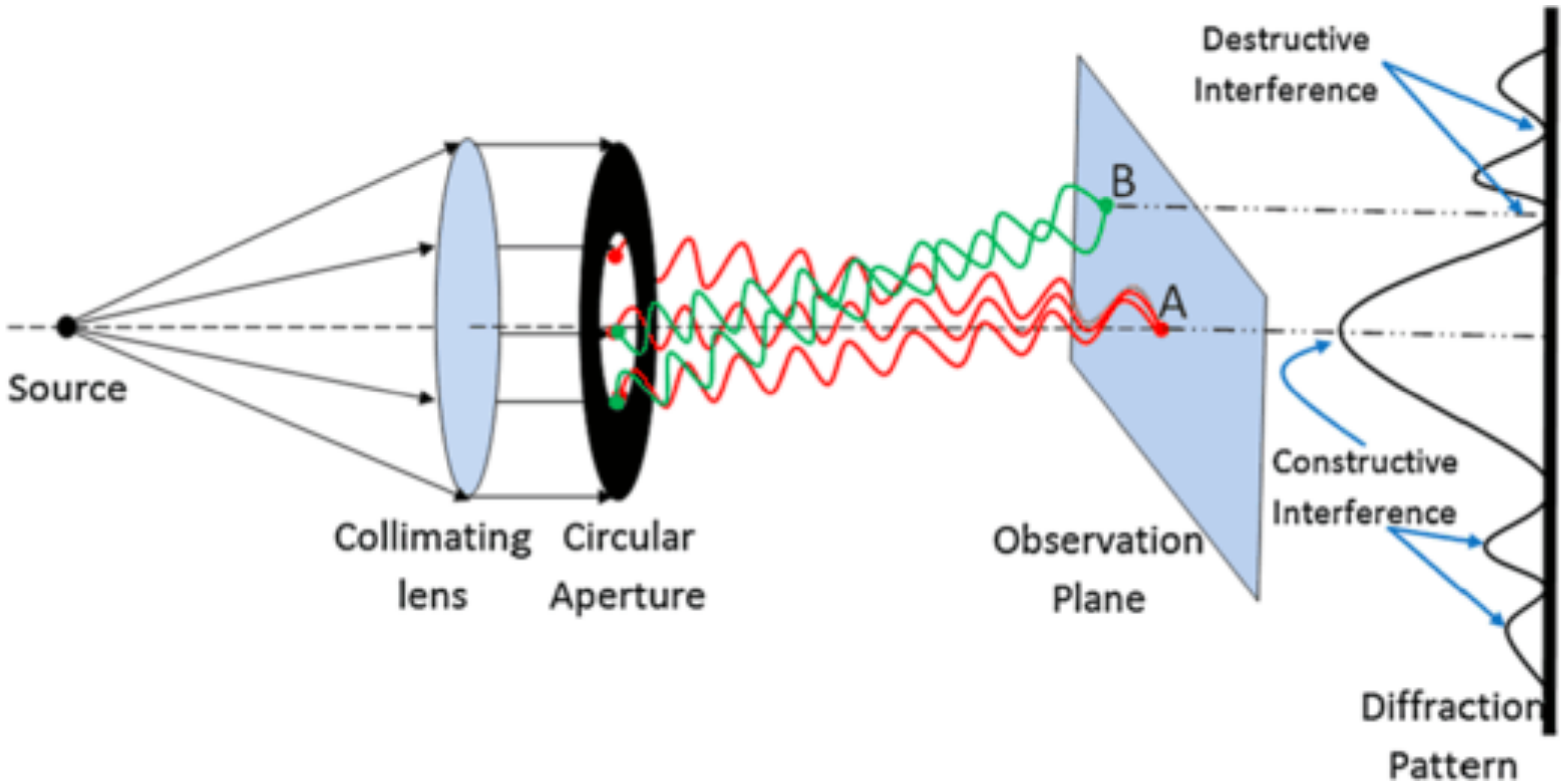
Radiometry and fundamental limits

- Radiometry: measuring the radiant flux (power) of electromagnetic radiation
  - Microwave radiometry: used to measure temperatures and properties of objects (astronomy and remote sensing)
- Fundamental limits in (classical) optics come from thermodynamics and the wave nature of light
  - Brightness is conserved
  - Diffraction limits the resolution, which depends on the size of the aperture
  - Etendue/throughput/AΩ is conserved, and also depends on size of aperture

Telescope Design

- Build the largest telescope you can afford
- Provide diffraction-limited images over as large of an area as we can cover with detectors
- Design it to be efficient
- Shield the signal from unwanted contamination
- Adjust the final beam to match the signal optimally onto detectors

Image formation



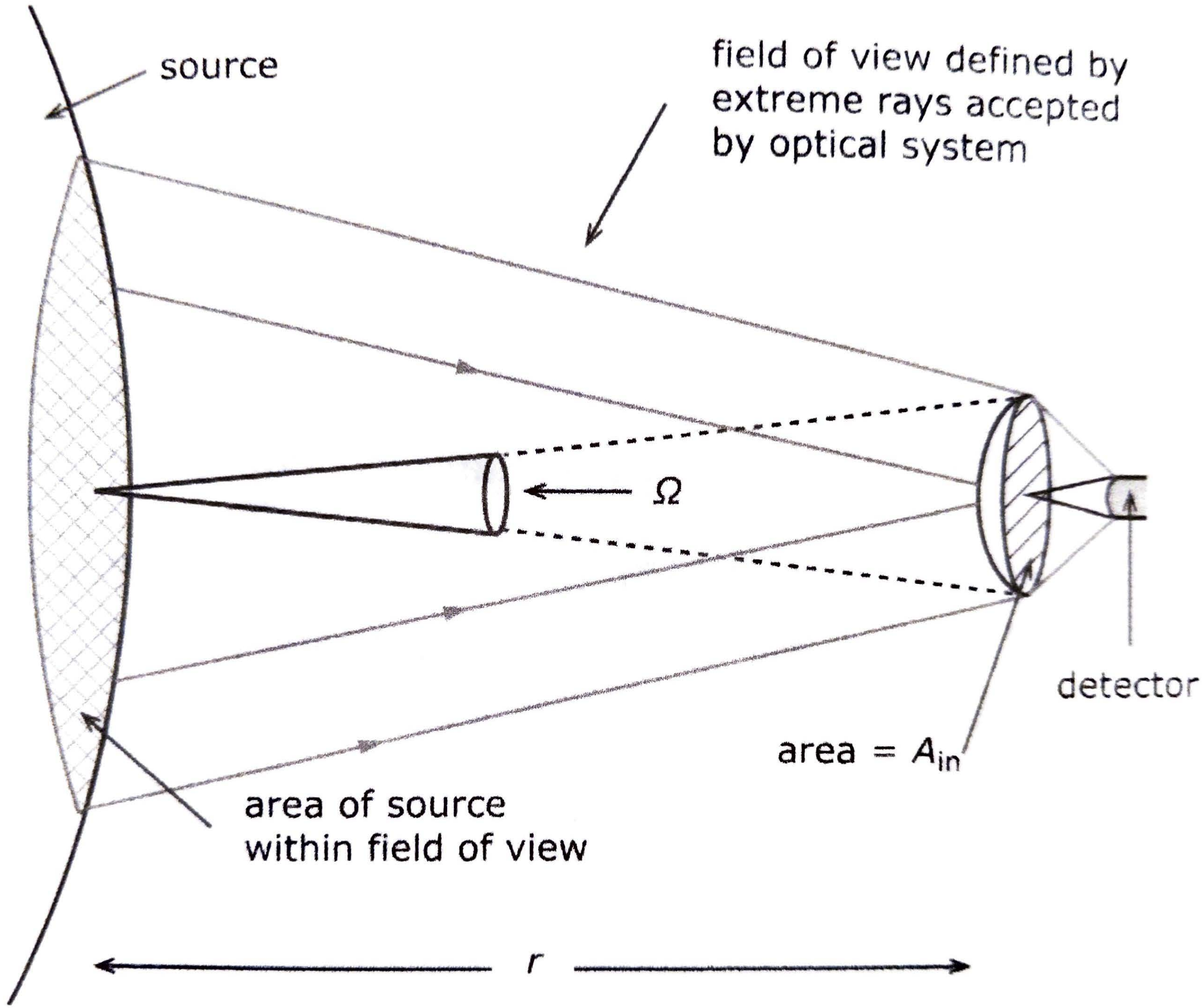
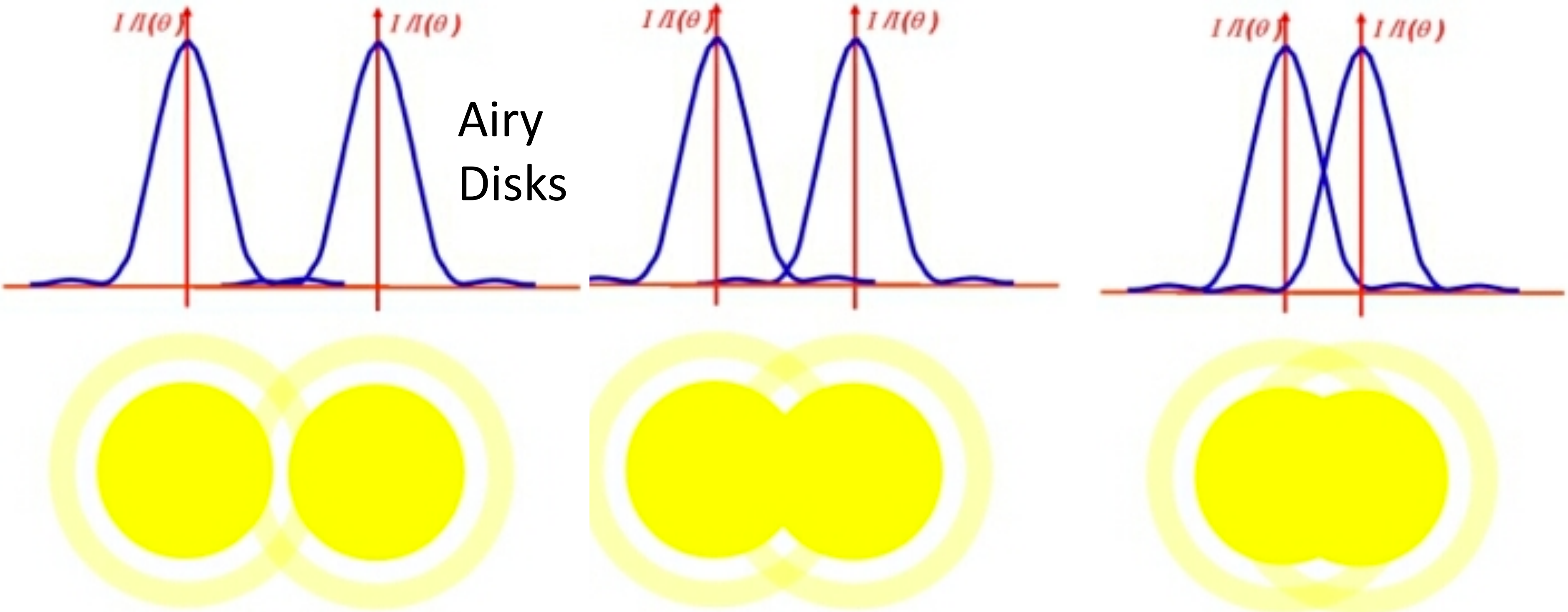


Figure 1.4. Geometry for detected signals.

Resolving limit (Rayleigh Criterion)



http://astronomy.swin.edu.au/cosmos/R/Rayleigh+Criterion

Telescope mounts

- Historically, telescopes were often on an equatorial mount
  - One axis is aligned with a celestial pole, making it much easier to track celestial objects
  - Amateur telescopes still commonly use this kind
- Modern telescopes are typically on a computer-controlled altitude-azimuth (alt-az) mount

Telescope parameters

- Focal length **f** – measured by projecting rays from focus back to match the diameter of the aperture
- **f-number** aka f-ratio aka focal ratio F = focal length/aperture
  - F = f/D
  - Dimensionless quantity, usually written like “f/10”
  - Energy per unit time onto a single pixel is proportional to (f/D)^-2
  - Large f-number: “slow”
    - High magnification, but lower brightness (longer exposure time needed)
  - Small f-number: “fast”
    - Shorter exposure time needed
- “effective focal length” = Magnification * f_primary
- For example, a telescope can be spec’d by its primary mirror size, its focal length, and the f-ratio of its secondary

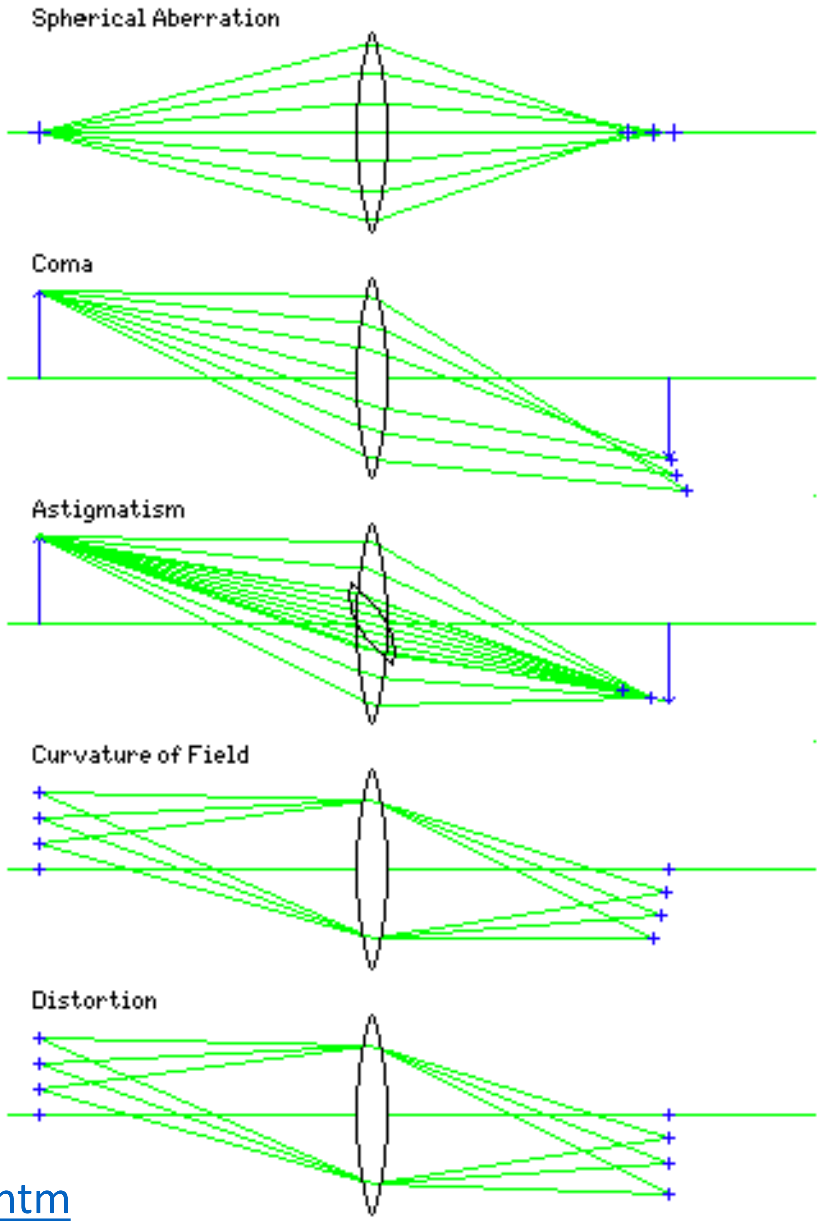
Telescope parameters

- **Plate scale** – how to translate physical units at the focal plane to projected area on the sky
  - How big will this 1 arcmin object appear on my sensor?
  - “Magnification” depends on focal length of primary mirror, and effective focal length of secondary (or eyepiece)
    - For an amateur telescope, M = f_primary/f_eyepiece
  - f_equivalent = M * f_primary
  - Probably know f_primary and F_secondary=f_equivalent/D
  - M = F_secondary * D / f_primary
  - See lecture notes

Telescope Parameters

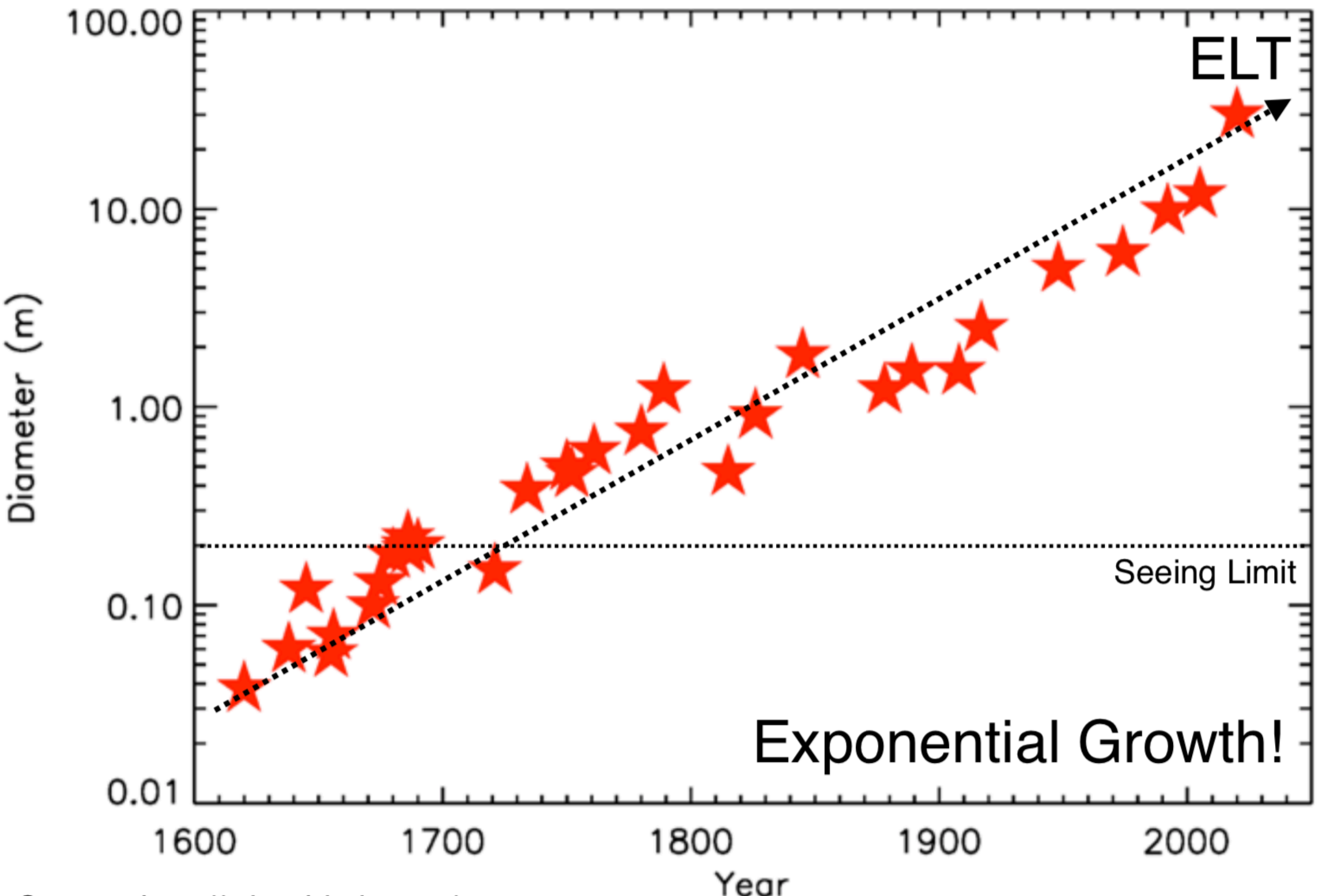
- **Field of view**: total angle on the sky that can be imaged by the telescope
- **Stop**: a physical mechanism to limit the bundle of light that can pass through
    - **Aperture stop**: limits the incoming bundle of rays
        - E.g. the edge of the primary mirror
    - **Field stop**: Limits the range of angles that the telescope can accept (limits the field of view)
- **Pupil**: an image of the aperture stop (or primary mirror)
    - Entrance pupil: ahead of stop
    - Exit pupil: behind stop

Primary Aberrations

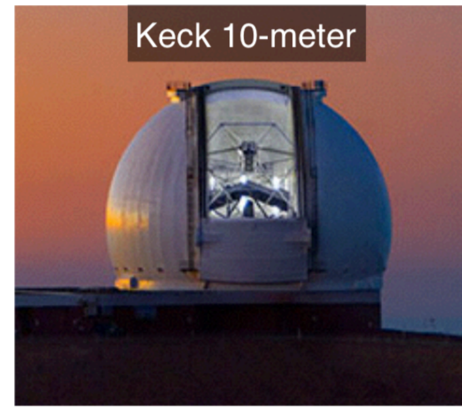


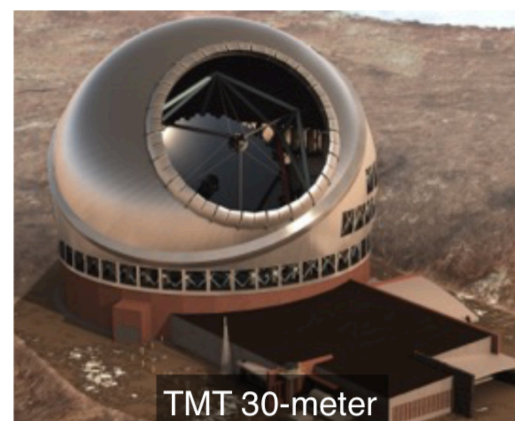
http://www.quadibloc.com/science/opt0505.htm

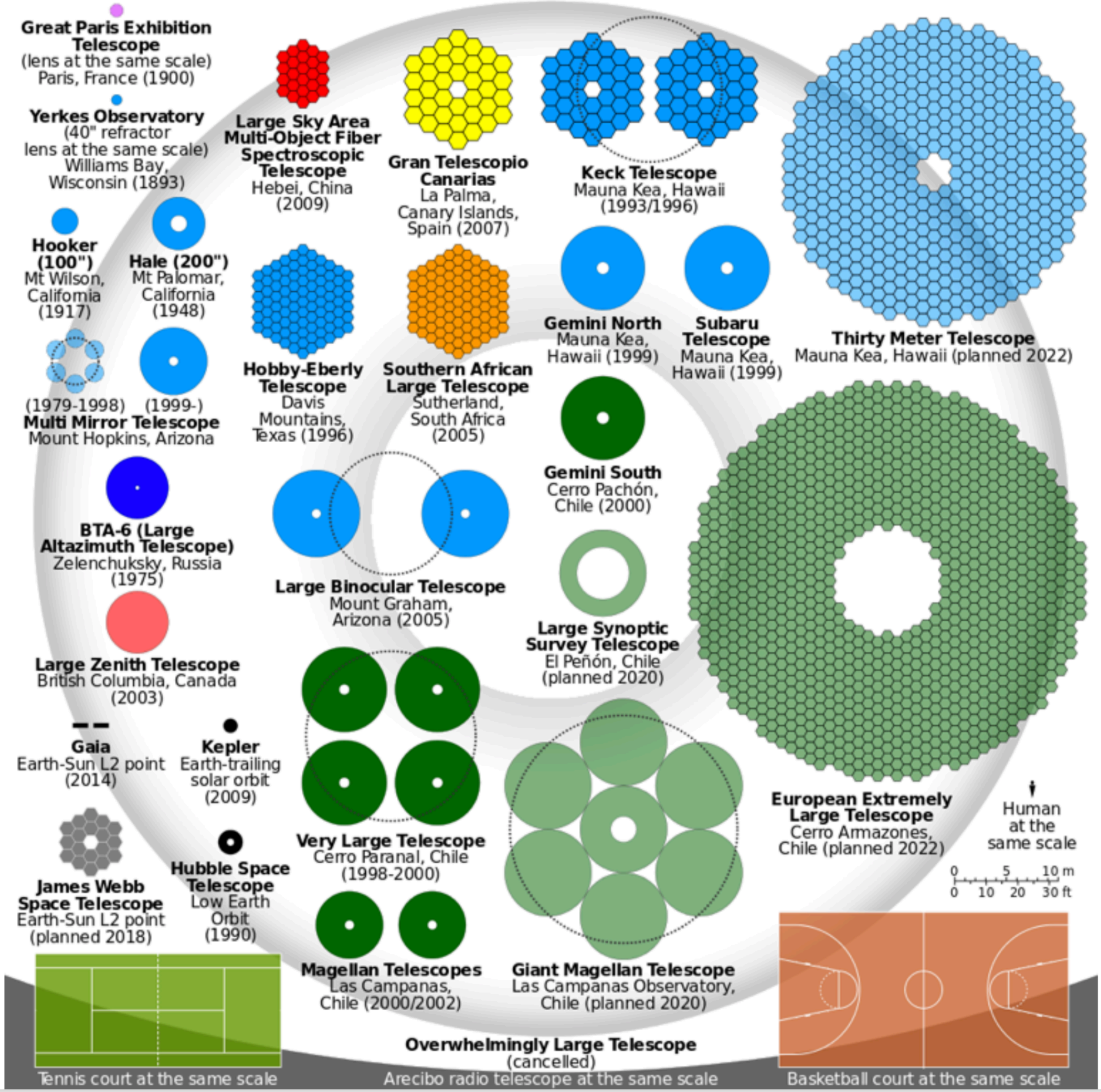
Explosive Growth in Ground-Based Optical/Infrared Telescopes



Source: http://stjarnhimlen.se/






Great Paris Exhibition Telescope
(lens at the same scale)
Paris, France (1900)
Yerkes Observatory
(40" refractor
lens at the same scale)
Williams Bay,
Wisconsin (1893)
Large Sky Area Multi-Object Fiber Spectroscopic Telescope
Hebei, China
(2009)
Gran Telescopio Canarias
La Palma,
Canary Islands,
Spain (2007)
Keck Telescope
Mauna Kea, Hawaii
(1993/1996)
Hooker (100")
Mt Wilson,
California
(1917)
Hale (200")
Mt Palomar,
California
(1948)
Hobby-Eberly Telescope
Davis Mountains,
Texas (1996)
Southern African Large Telescope
Sutherland,
South Africa
(2005)
Gemini North
Mauna Kea,
Hawaii (1999)
Subaru Telescope
Mauna Kea,
Hawaii (1999)
Thirty Meter Telescope
Mauna Kea, Hawaii (planned 2022)
(1979-1998)
(1999-)
Multi Mirror Telescope
Mount Hopkins, Arizona
Gemini South
Cerro Pachón,
Chile (2000)
BTA-6 (Large Altazimuth Telescope)
Zelenchuksky, Russia
(1975)
Large Binocular Telescope
Mount Graham,
Arizona (2005)
Large Synoptic Survey Telescope
El Peñón, Chile
(planned 2020)
Large Zenith Telescope
British Columbia, Canada
(2003)
Gaia
Earth-Sun L2 point
(2014)
Kepler
Earth-trailing
solar orbit
(2009)
European Extremely Large Telescope
Cerro Armazones,
Chile (planned 2022)
Human
at the
same scale
James Webb Space Telescope
Earth-Sun L2 point
(planned 2018)
Hubble Space Telescope
Low Earth
Orbit
(1990)
Very Large Telescope
Cerro Paranal, Chile
(1998-2000)
0 5 10 m
0 10 20 30 ft
Magellan Telescopes
Las Campanas,
Chile (2000/2002)
Giant Magellan Telescope
Las Campanas Observatory,
Chile (planned 2020)
Overwhelmingly Large Telescope
(cancelled)
Tennis court at the same scale
Arecibo radio telescope at the same scale
Basketball court at the same scale

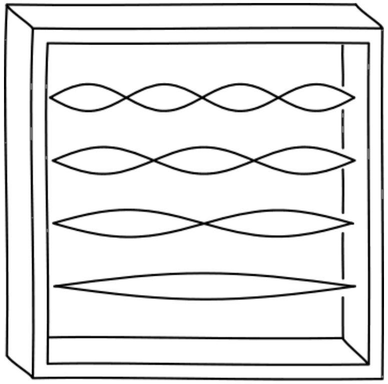
Optical Telescope Examples

- **Thirty Meter Telescope**
- **Hobby-Eberly Telescope**
- **Large Synoptic Survey Telescope**
- **Gaia**
- **James Webb Space Telescope**
- **Hale Telescope**
- **Keck Telescope**
- **Automated Planet Finder**
- **Transiting Exoplanet Survey Satellite (TESS)**
- **Kepler**
- **The Leviathan of Parsonstown**
- **Hooker Telescope**

Photon statistics

- Photons are bosons, and they follow Bose-Einstein statistics
  - Arrivals are **not independent**
  - Noise is not just proportional to number of photons received
  - Two noise terms: **shot noise** and **photon bunching/wave noise**
- Boltzmann occupation number n_s
  - number of photons in standing-wave mode in box at temperature T
  - number of photons/s/Hz in (diffraction limited) beam in free space (Richards 1994, J.Appl.Phys)

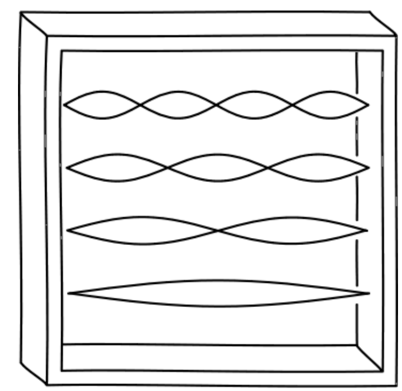
$$\langle n_s \rangle = \left(e^{h\nu_s/kT} - 1\right)^{-1}$$



Photon statistics

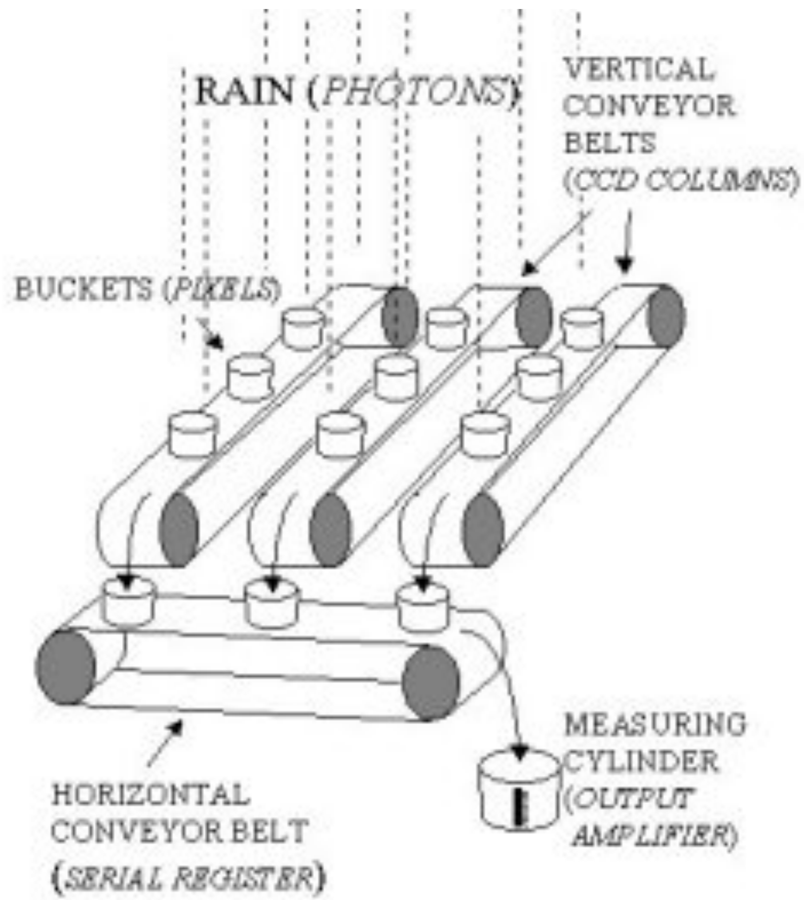
- Three regimes
  - $n_s >> 1$
    - $hv << kT$
    - Radio wavelengths
    - Photon "bunching" is significant
  - $n_s \sim 1$
    - $hv \sim kT$
    - ~ Millimeter wavelengths
    - Both noise terms must be considered
  - $n_s << 1$
    - $hv >> kT$
    - Shot noise is significant
    - **Noise follows Poisson statistics**

$$\langle n_s \rangle = \left(e^{h\nu_s/kT} - 1\right)^{-1}$$



Many slides/animations borrowed from CCD Primer

http://www.ing.iac.es/~eng/detectors/CCD_Info/CCD_Primer.htm



Measurement Errors

- Accuracy
  - How close a result comes to the true value
- **Precision**
  - **Closeness of multiple measurements to each other**
- **Random error**
  - **Caused by uncontrollable fluctuations in the observations (for example, photon noise)**
- Systematic error
  - Non-statistical error caused by instrument or dataset (for example bias, faulty calibration)

Error propagation

- If we have n independent estimates Xj, each with an associated error σj, then the error is estimated as the weighted mean:
  - $\overline{X_w} = \sum_{j=1}^{n} w_j \bar{X}_j / \sum_{j=1}^{n} w_j \quad , w_j = 1/\sigma_j^2$
- The variance is given by
  - $\sigma_w^2 = 1/\sum_{j=1}^{n} 1/\sigma_j^2$
- The simplest case: if errors are Gaussian and **uncorrelated**, we can just add each error source in quadrature
  - $\sigma_{total}^2 = \ \sigma_a^2 + \sigma_b^2 + \sigma_c^2 + \sigma_d^2$

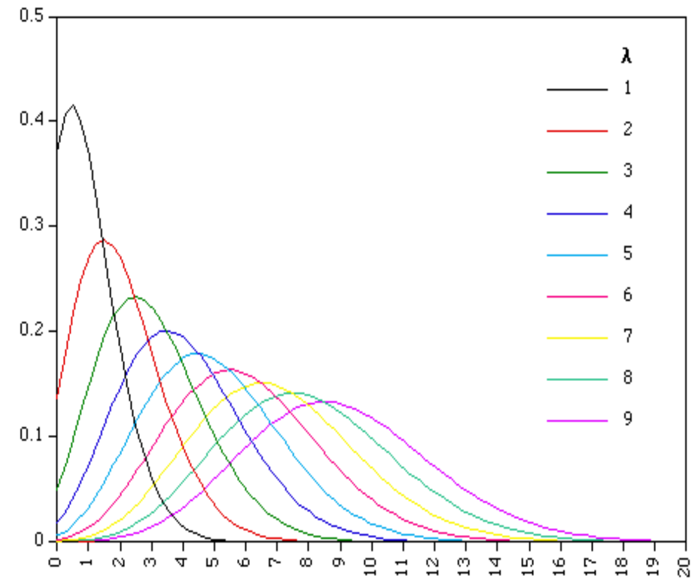
Statistics

- Statistics summarize and describe data
  - When data nicely follow a distribution, statistics are extremely useful in describing them
- Common statistical measures
  - Mean/average: $\bar{X} = \frac{1}{N}\sum_{i}^{N} X_i$
  - Median: Arrange values in order, and median is the (N/2 + 0.5) value (if N is odd) or median = $\frac{1}{2}\left(X_{\frac{N}{2}} + X_{\frac{N}{2}+1}\right)$ if N is even
  - Mode: most frequently occurring or most probable value
  - Mean Deviation: $\overline{\Delta X} = \frac{1}{N}\sum_{i=1}^{N} |X_i - X_{med}|$
  - Mean squares deviation: $S^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2$
  - Root-mean-square deviation: S
  - Order statistics (e.g. minimum, maximum, mode)

Distributions

- Distribution: expected behavior for a large number of independent measurements
- Mean and variance define the distribution function (as opposed to statistics that describe a dataset)
- Variance: $\sigma^2 \equiv \frac{1}{n}\sum(x_i - m)^2$
- Standard deviation: σ
- Poisson distribution
  - The binomial distribution in the limit of rare events (count-rate distribution)
    - Photon shot noise, radioactive decay
  - $f(x, \mu) = e^{-\mu}\mu^x/x!$
  - If μ photons are received per Δt, probability of receiving x photons in Δt is given by f(x, μ)
  - Mean: μ, variance: μ

Poisson distribution



Normal/Gaussian distribution

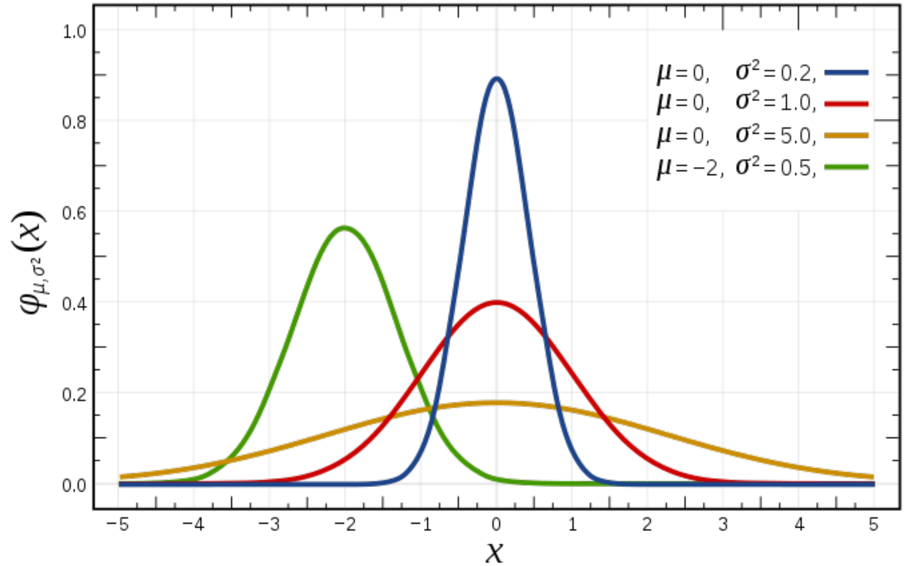
Distributions

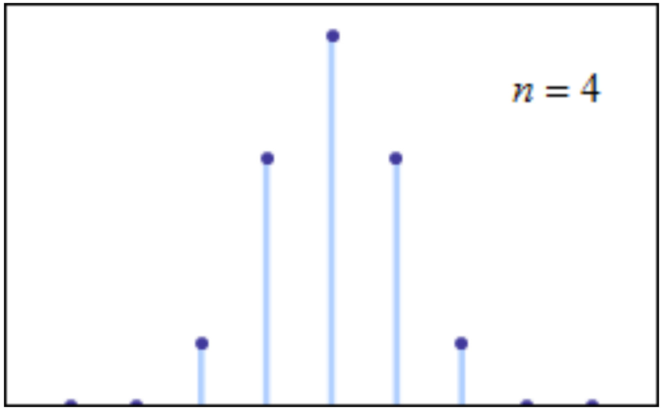
- Gaussian (normal) distribution
  - Mean and variance are independent

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

  - Poisson distribution is indistinguishable from Gaussian distribution for large μ

- **Central limit theorem**
  - Any random sampling of values will trend towards a Gaussian distribution as N gets very large
  - $\left[\frac{M_n-\mu}{\frac{\sigma}{\sqrt{n}}}\right] \rightarrow$ Gaussian distribution





https://en.wikipedia.org/wiki/File:De_moivre-laplace.gif

Detection significance

- Normal distribution gives the expected variation in background (non-signal) counts
- Detection significance is confidence in your measurement including some amount of signal
- For example, detecting a faint star

CCD Signal to Noise

- For light from a star hitting a single pixel
- $\frac{S}{N} = \frac{N_*}{\sqrt{N_* + N_{sky} + N_{dark} + N_{readout}^2}}$
- If star is bright (N_* is big), S/N will scale with square root of exposure time
- If $N_{readout}$ is big, S/N will scale linearly with exposure tiime