Astro 426/526

Fall 2019

Prof. Darcy Barron

Lecture 10: CCDs, statistics and error

Reminders

- Last week: detection principles, photon detectors
- This week: CCDs, then statistics, error analysis (and applications to detectors)
- Next week: review, and mid-term exam

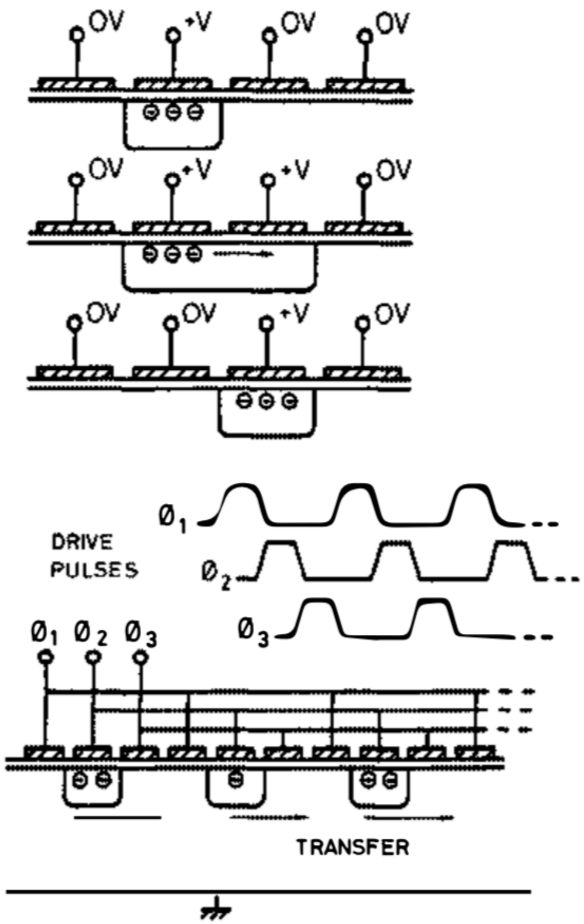


Figure 2a The charge-transfer mechanism employed in three-phase CCDs.

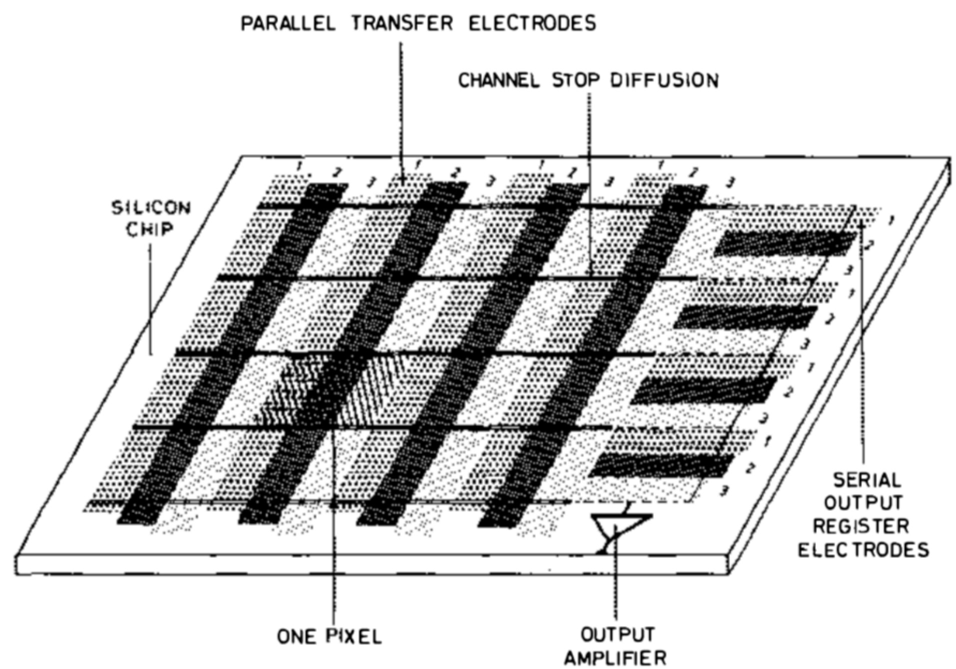
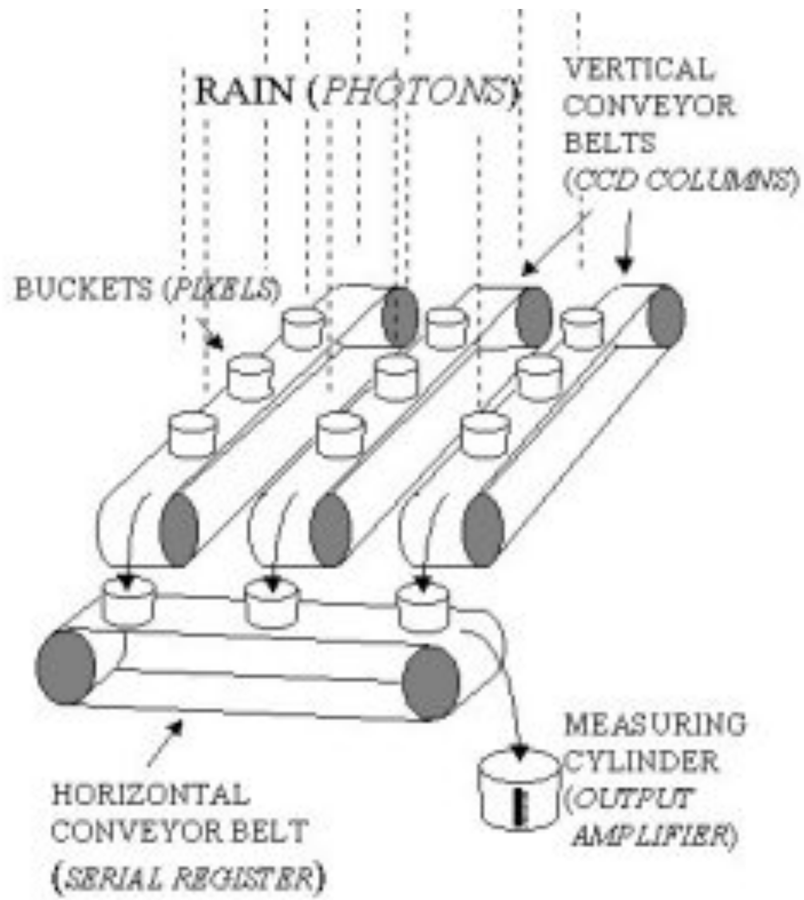


Figure 2b The basic layout of a three-phase two-dimensional CCD. The sequence 1, 2, 3 on each set of electrodes indicates the normal direction of charge transfer in the parallel and serial registers.

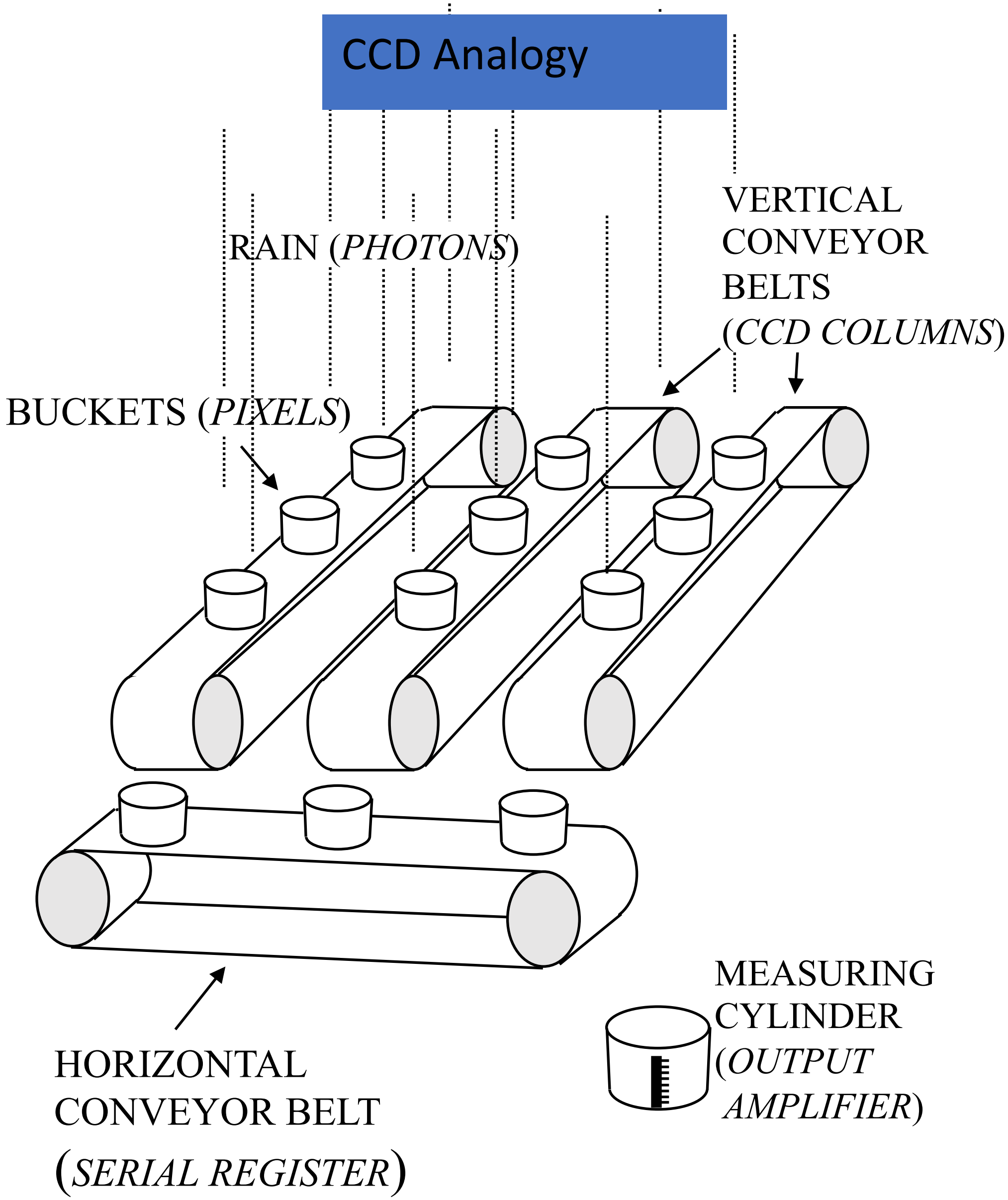
Charged-Coupled Devices in Astronomy by Craig D. Mackay
Ann. Rel. Astron. Astrophys. 1986.24:255-83

Many slides/animations borrowed from CCD Primer

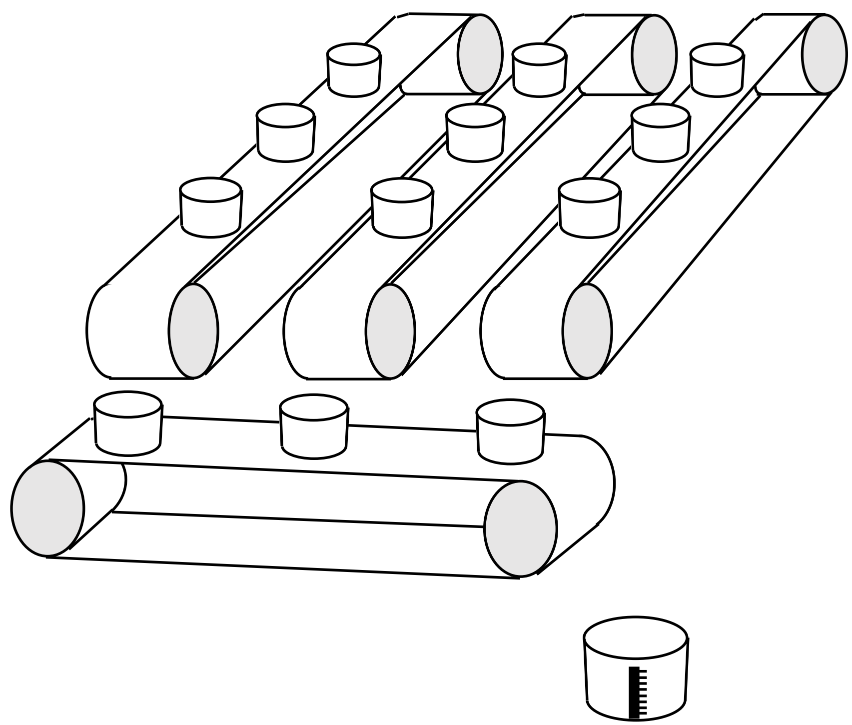
http://www.ing.iac.es/~eng/detectors/CCD_Info/CCD_Primer.htm



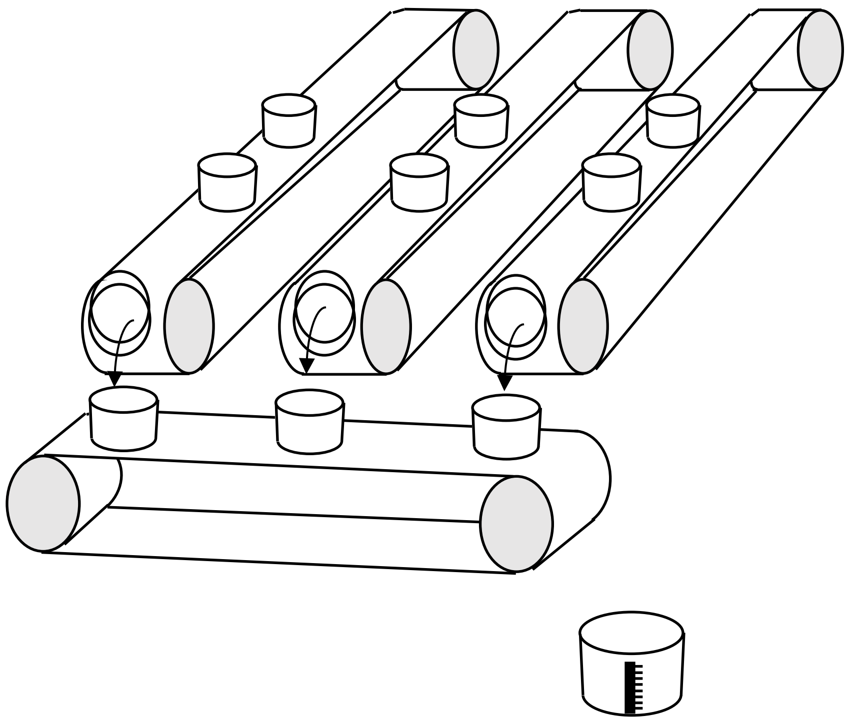
CCD Analogy


RAIN (*PHOTONS*)
VERTICAL CONVEYOR BELTS (*CCD COLUMNS*)
BUCKETS (*PIXELS*)
MEASURING CYLINDER (*OUTPUT AMPLIFIER*)
HORIZONTAL CONVEYOR BELT (*SERIAL REGISTER*)

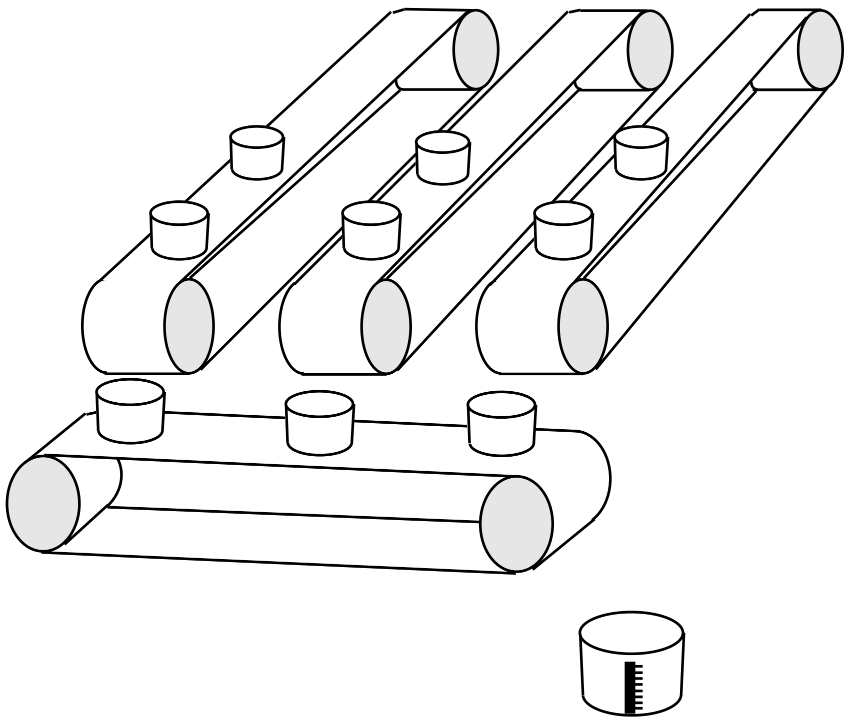
Exposure finished, buckets now contain samples of rain.



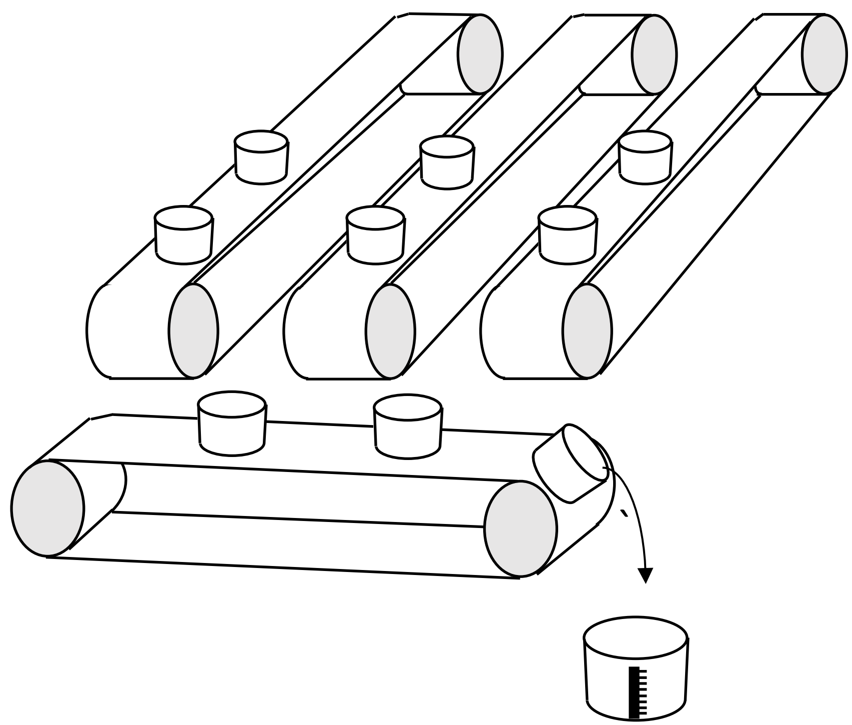
Conveyor belt starts turning and transfers buckets. Rain collected on the vertical conveyor is tipped into buckets on the horizontal conveyor.

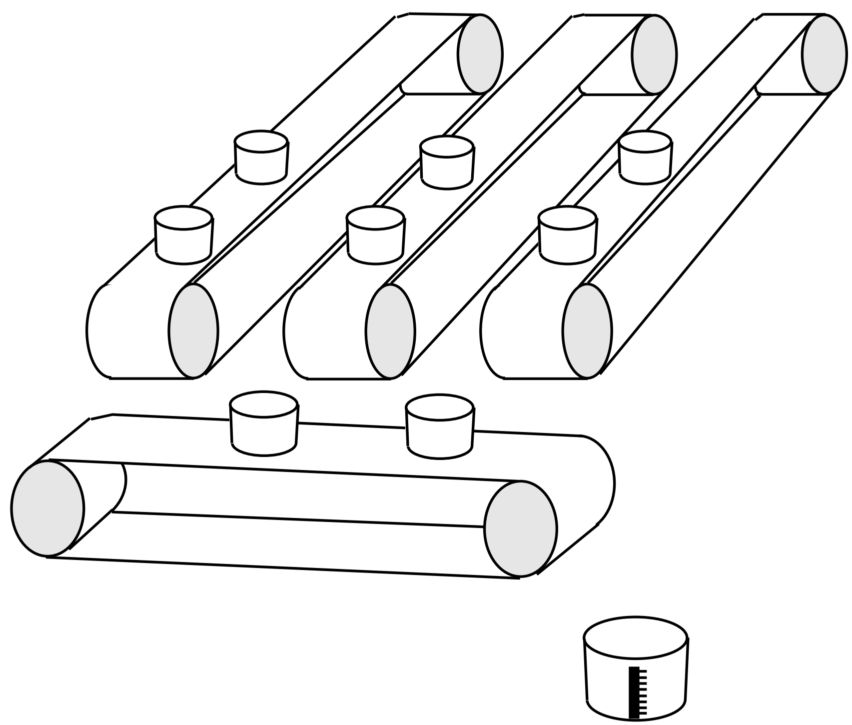


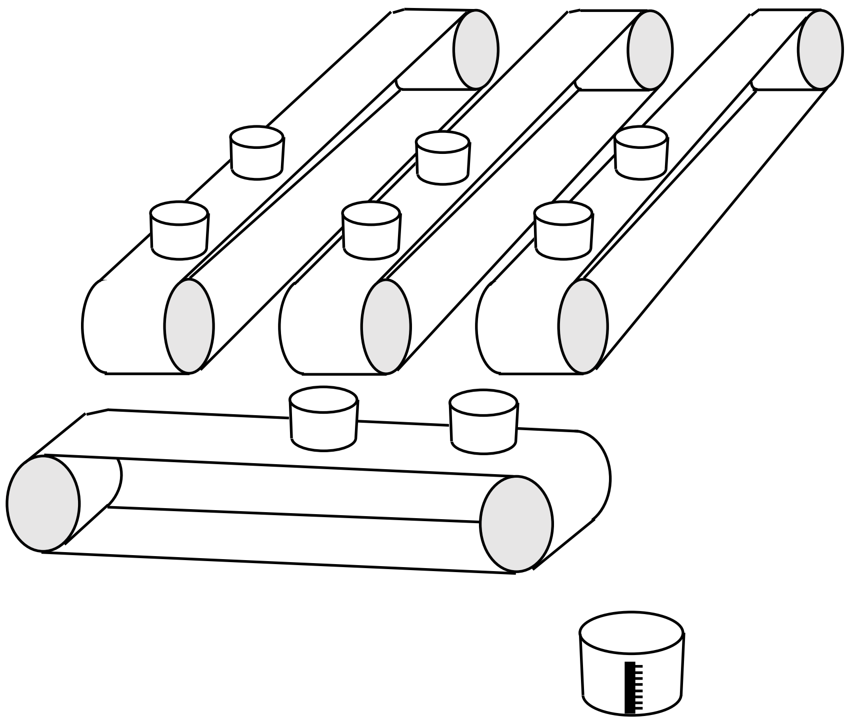
Vertical conveyor stops. Horizontal conveyor starts up and tips each bucket in turn into the measuring cylinder .

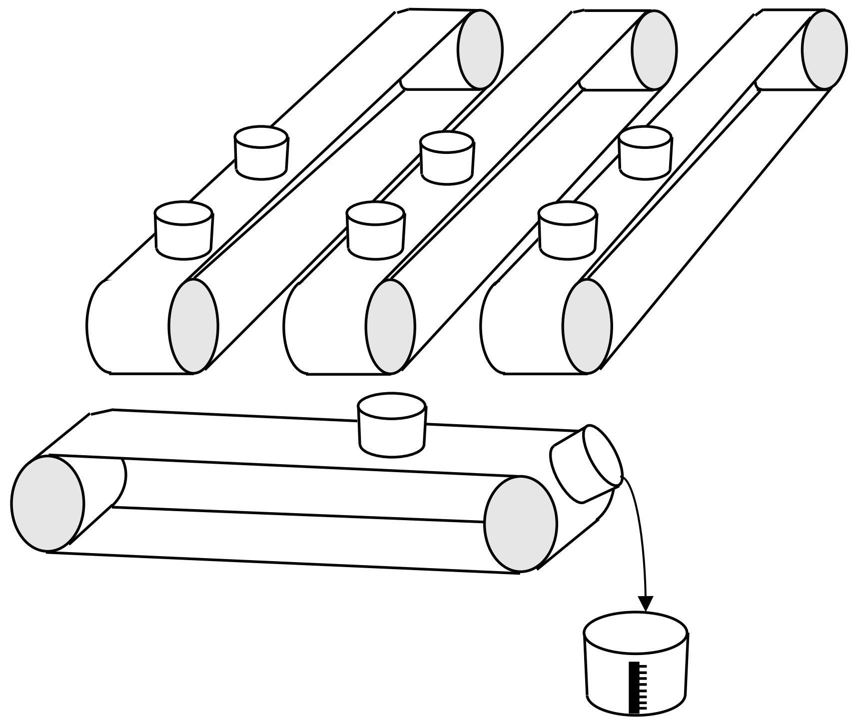


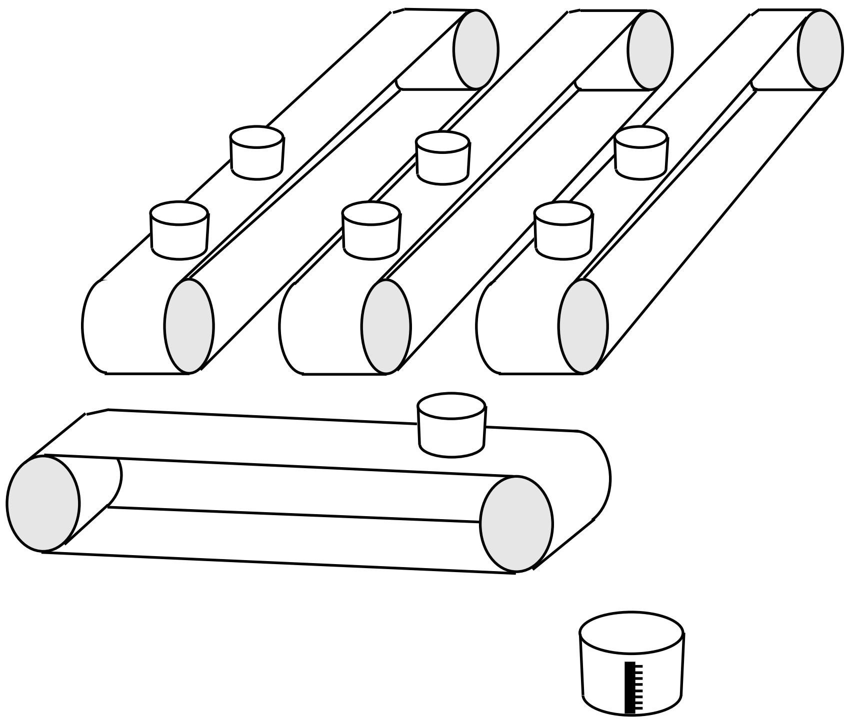
After each bucket has been measured, the measuring cylinder is emptied , ready for the next bucket load.

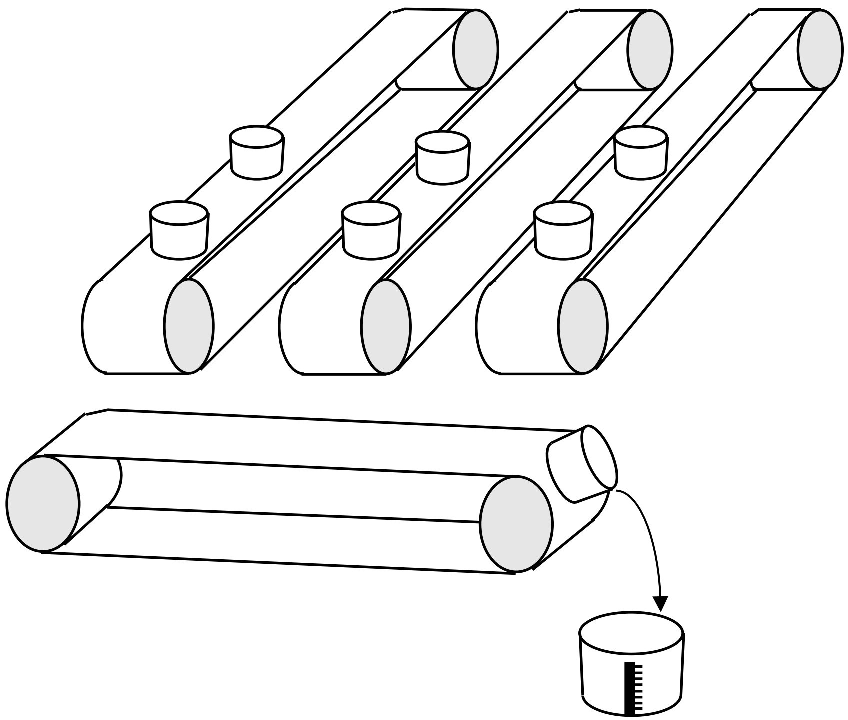


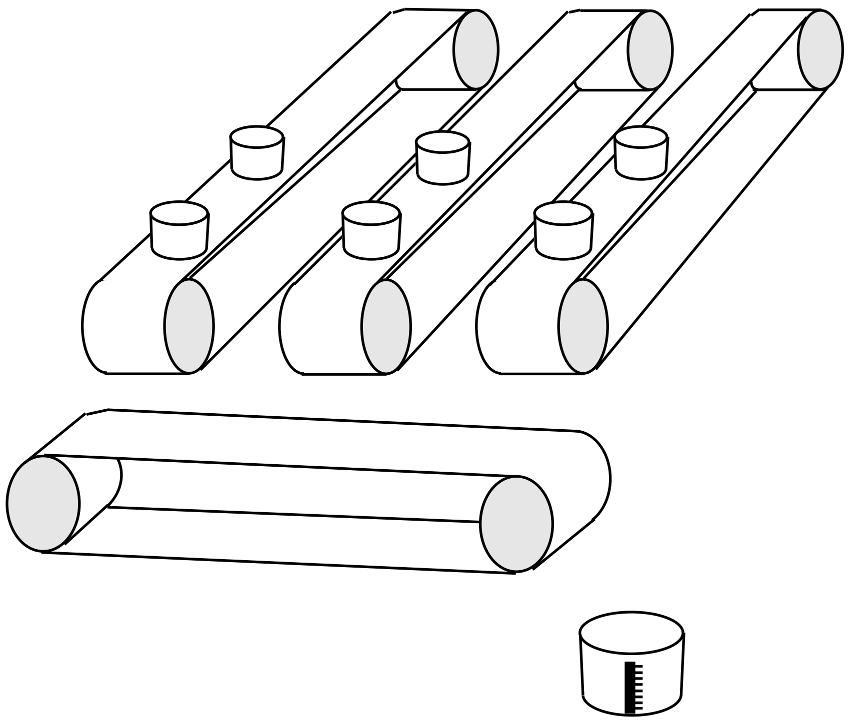




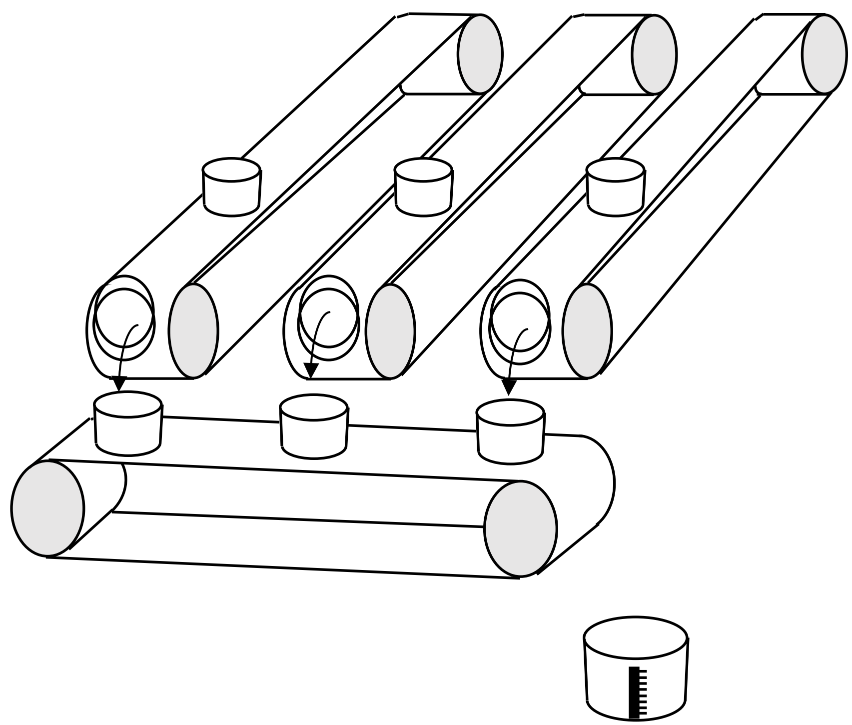


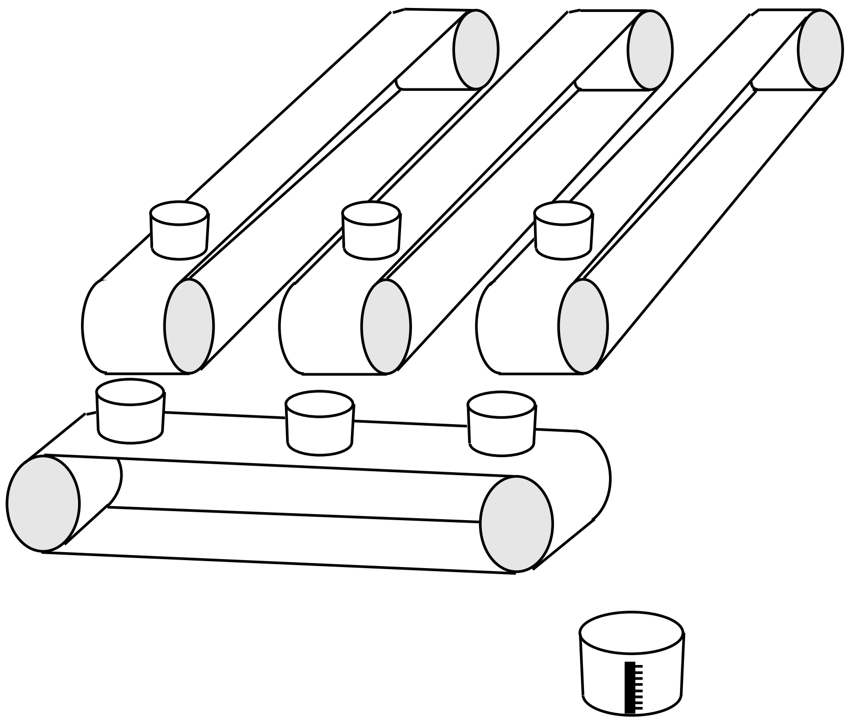


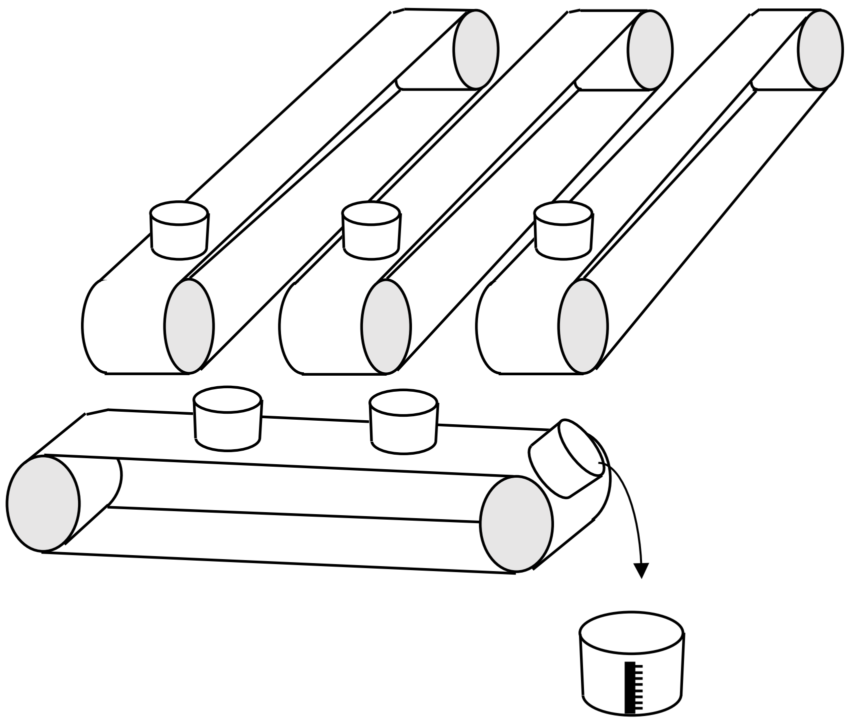


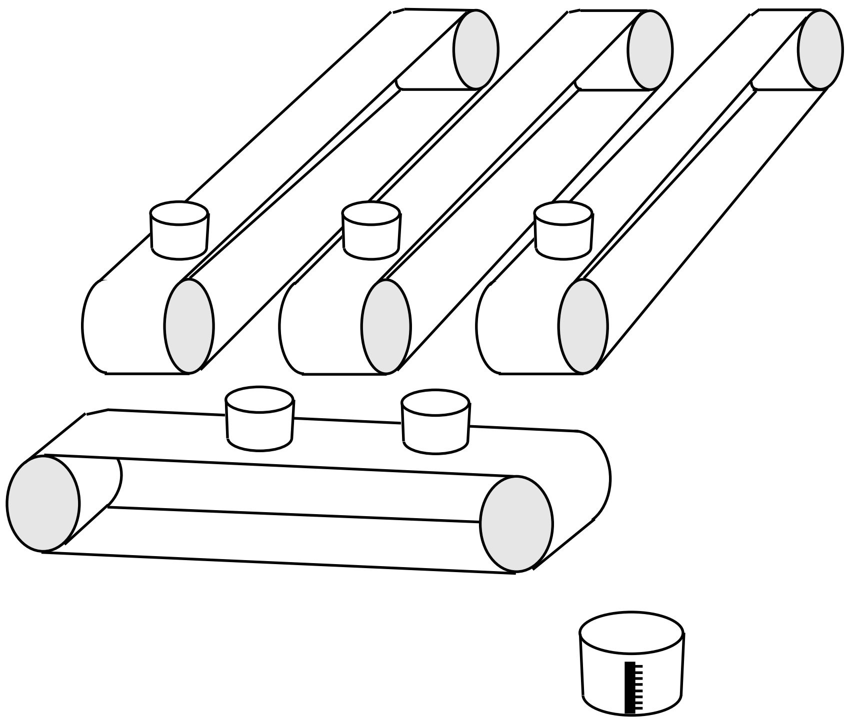


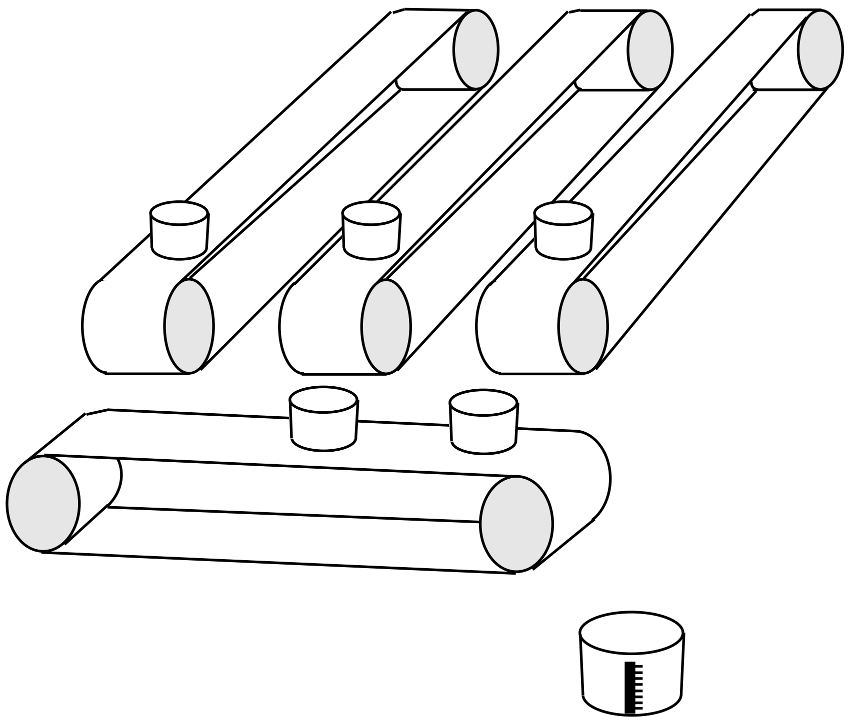
A new set of empty buckets is set up on the horizontal conveyor and the process is repeated.

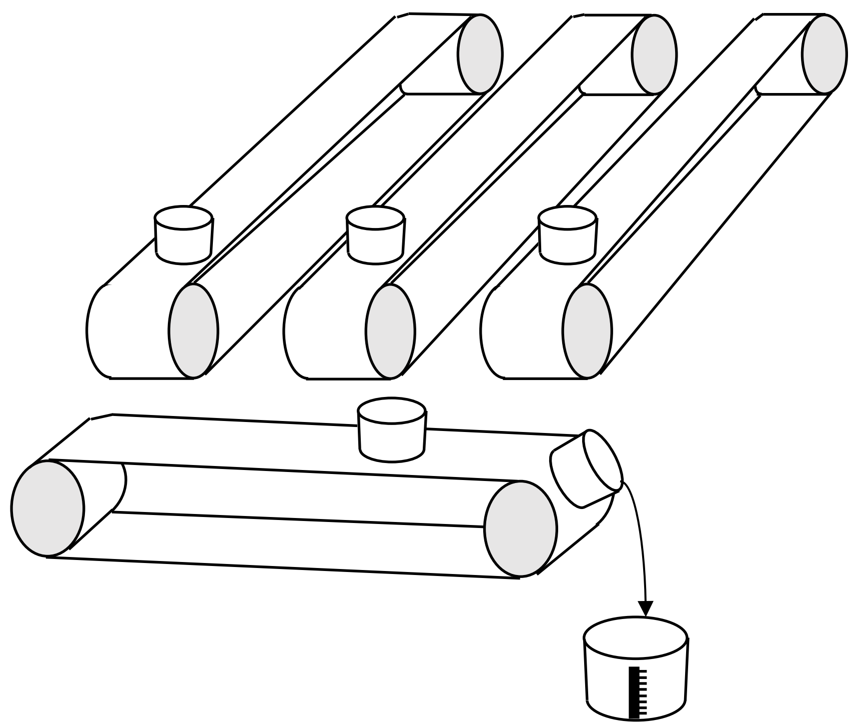


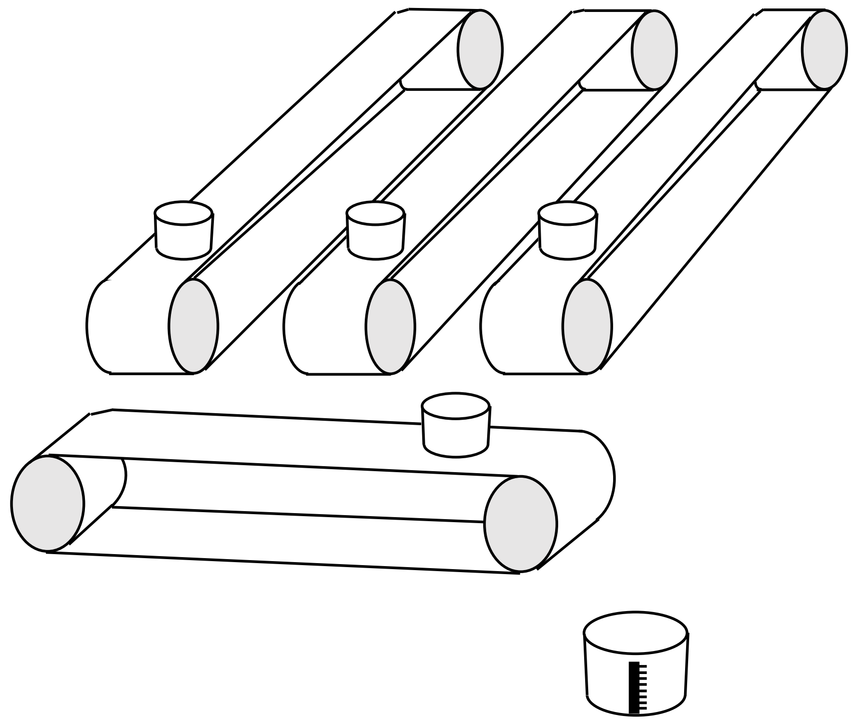


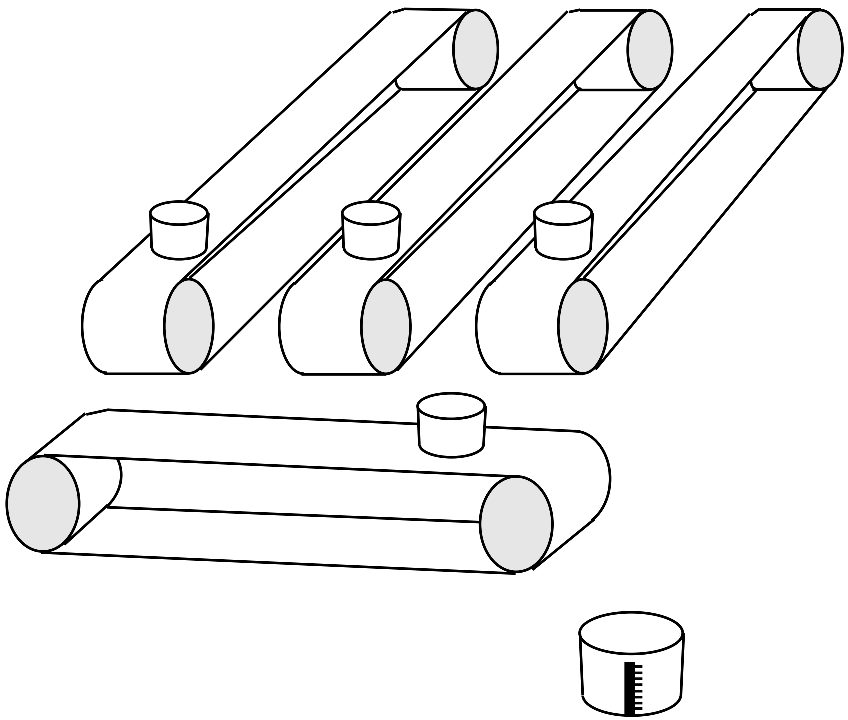


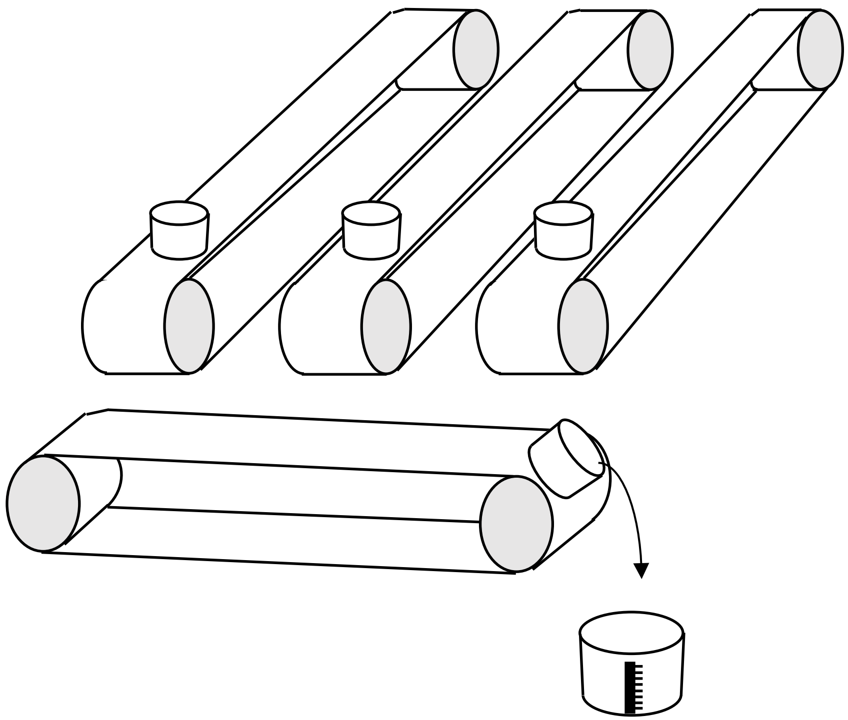


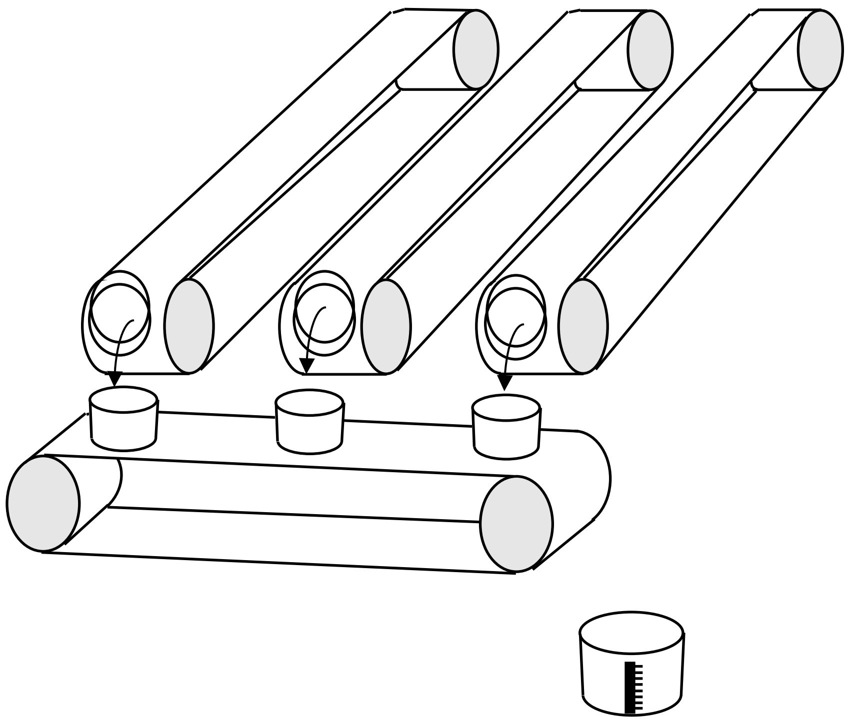


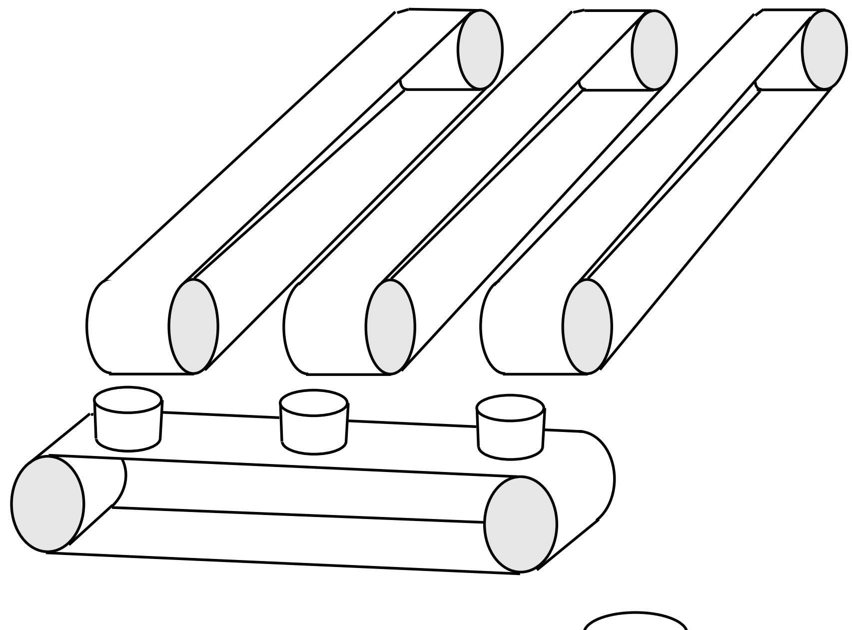


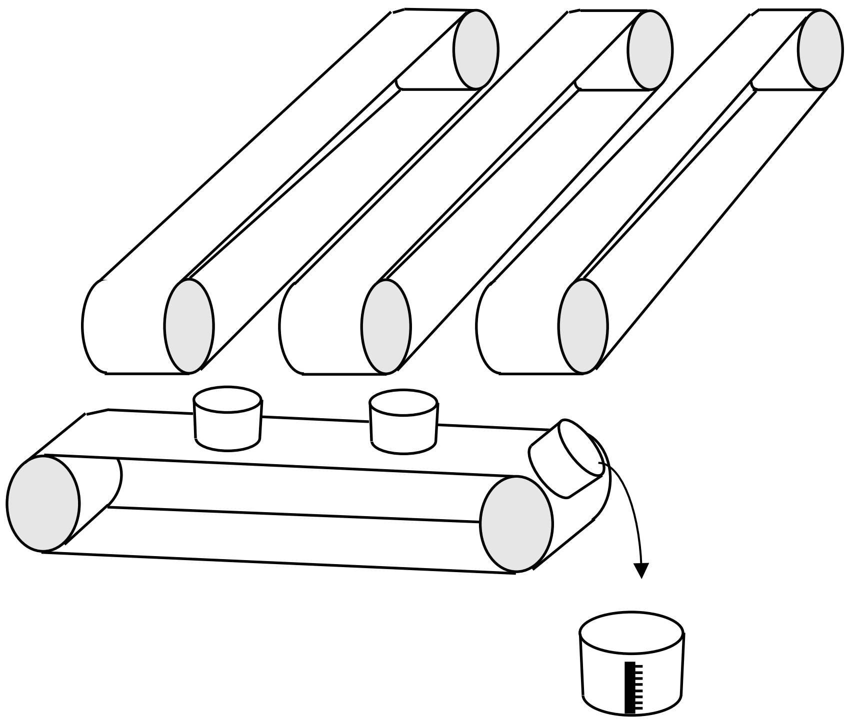


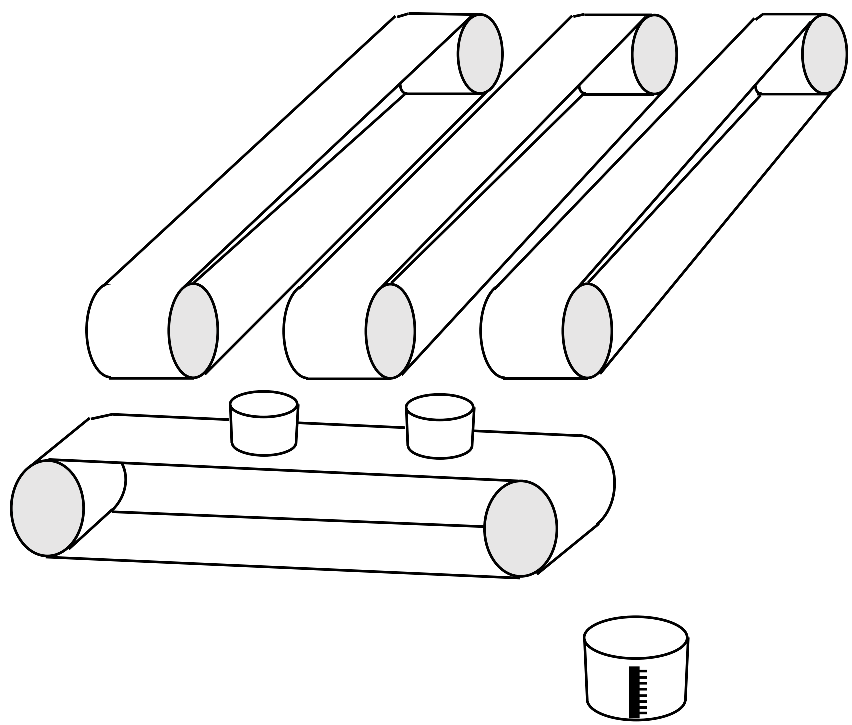


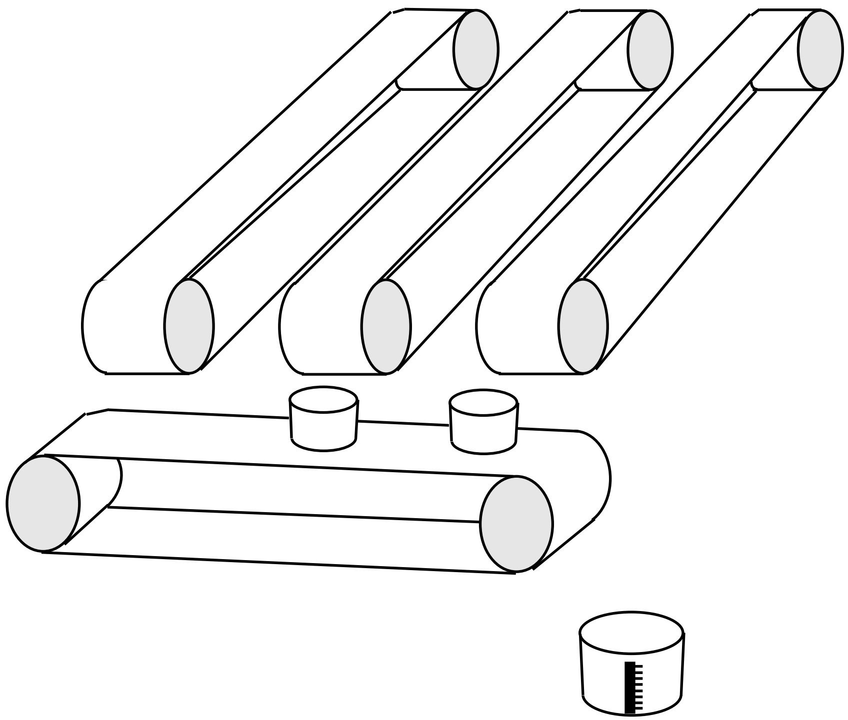


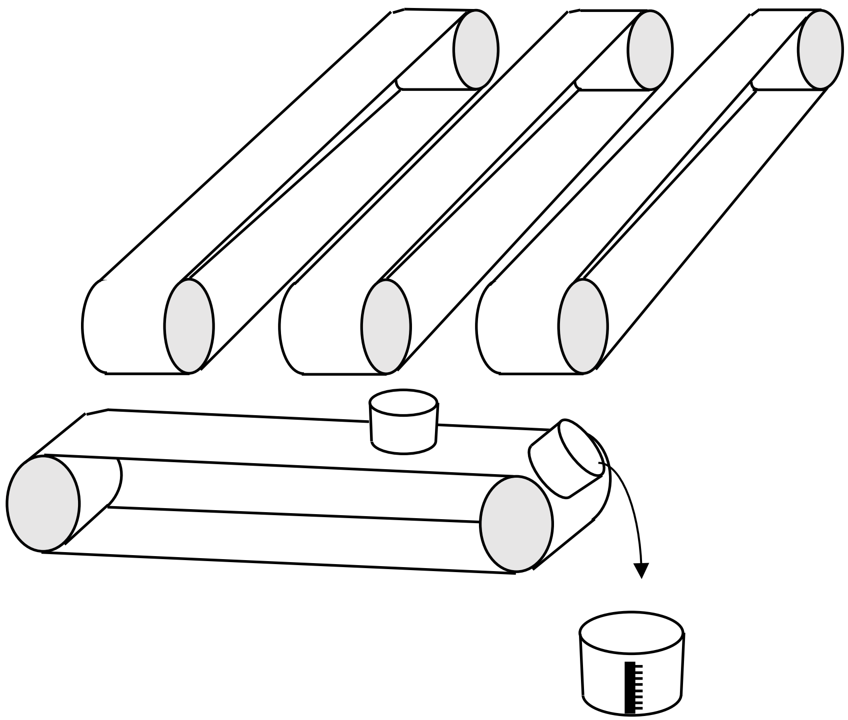


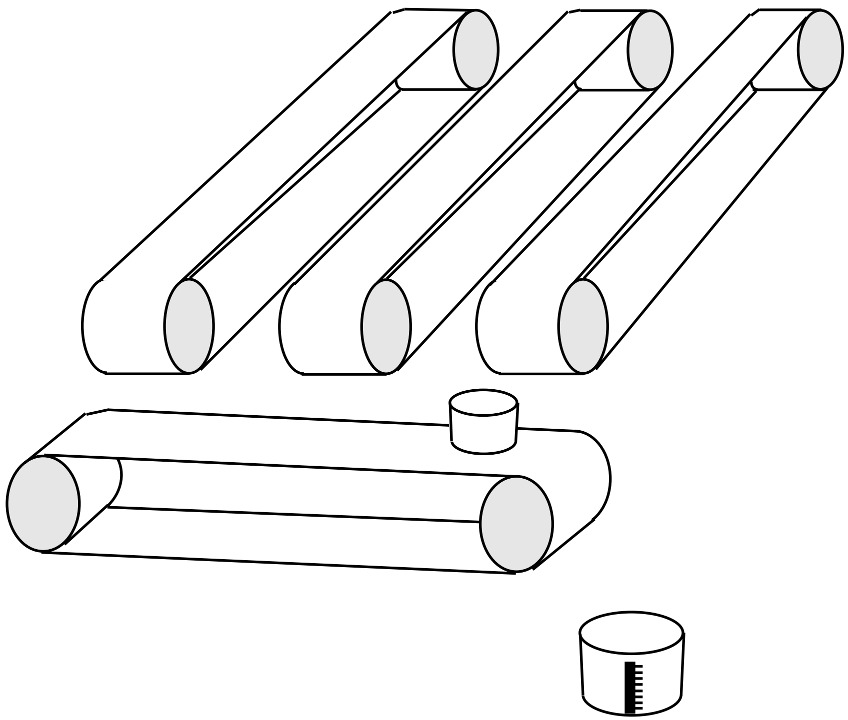


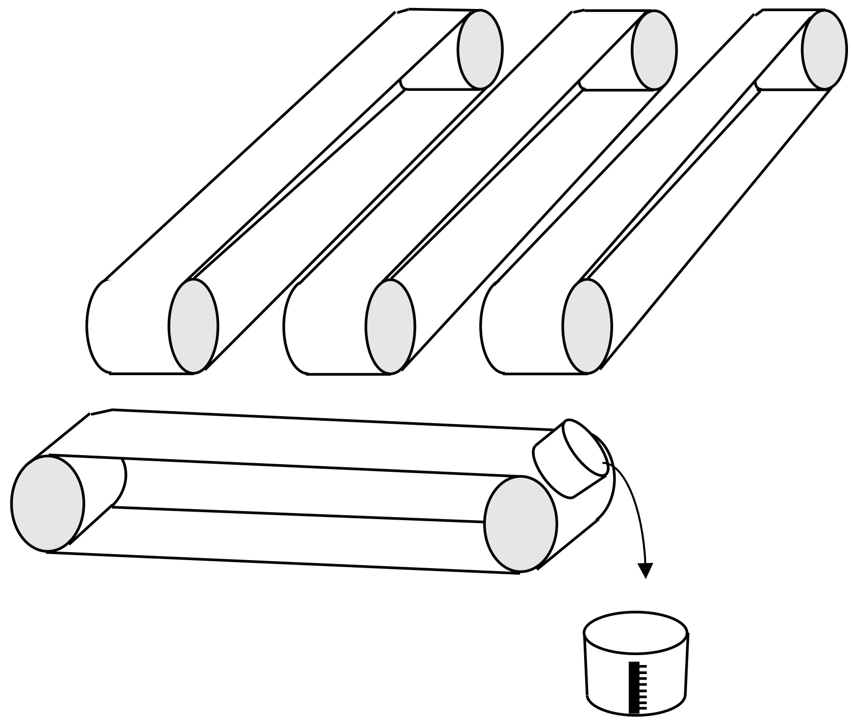




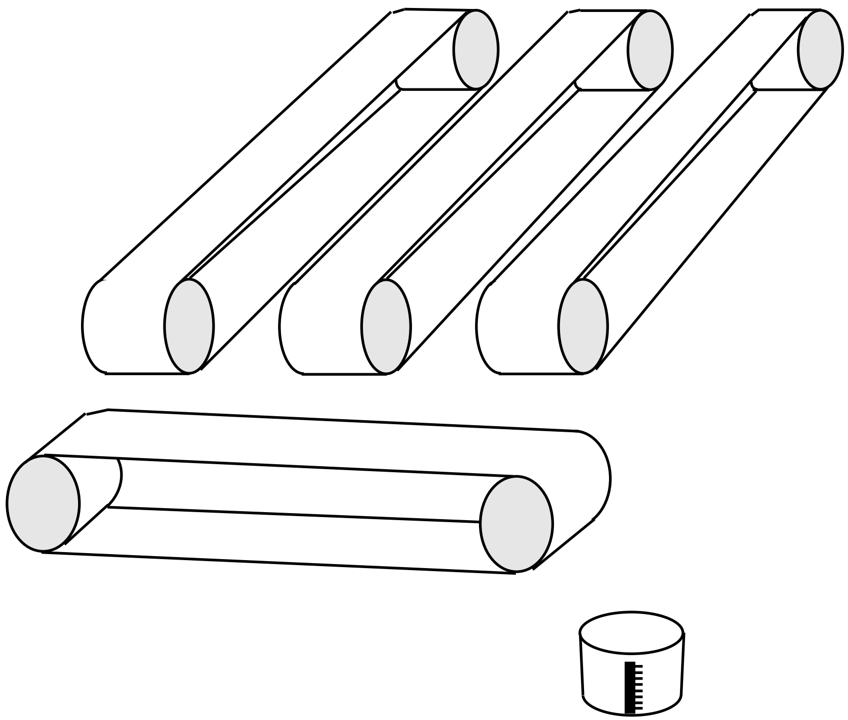








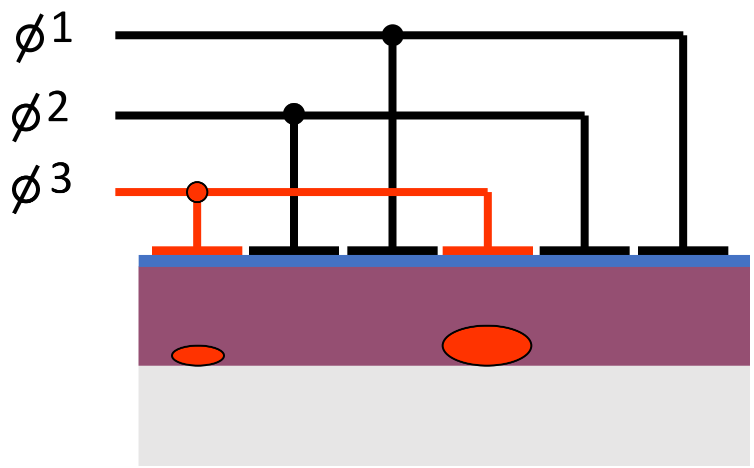
Eventually all the buckets have been measured, the CCD has been read out.



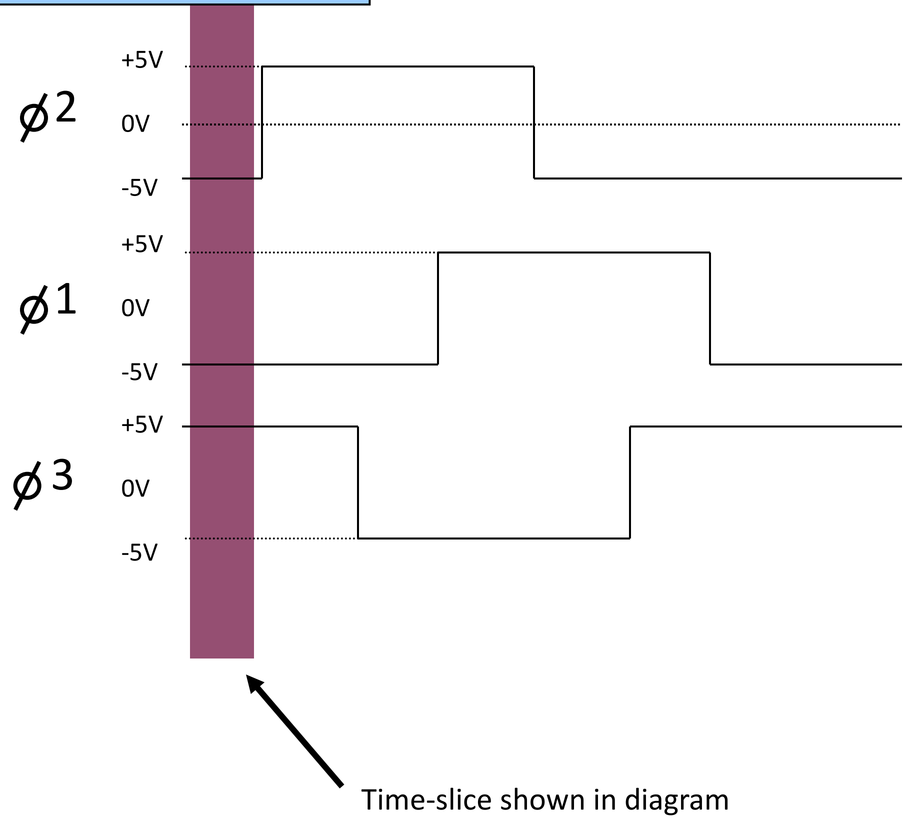
Charge Transfer in a CCD 1.

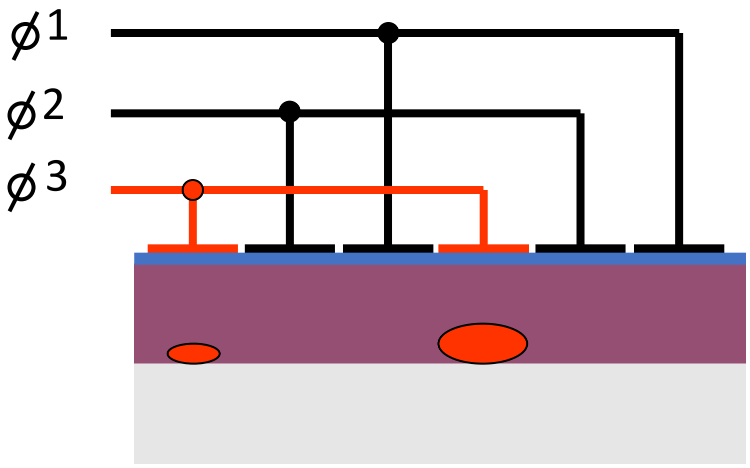
In the following few slides, the implementation of the ‘conveyor belts’ as actual electronic structures is explained.

The charge is moved along these conveyor belts by modulating the voltages on the electrodes positioned on the surface of the CCD. In the following illustrations, electrodes colour coded red are held at a positive potential, those coloured black are held at a negative potential.

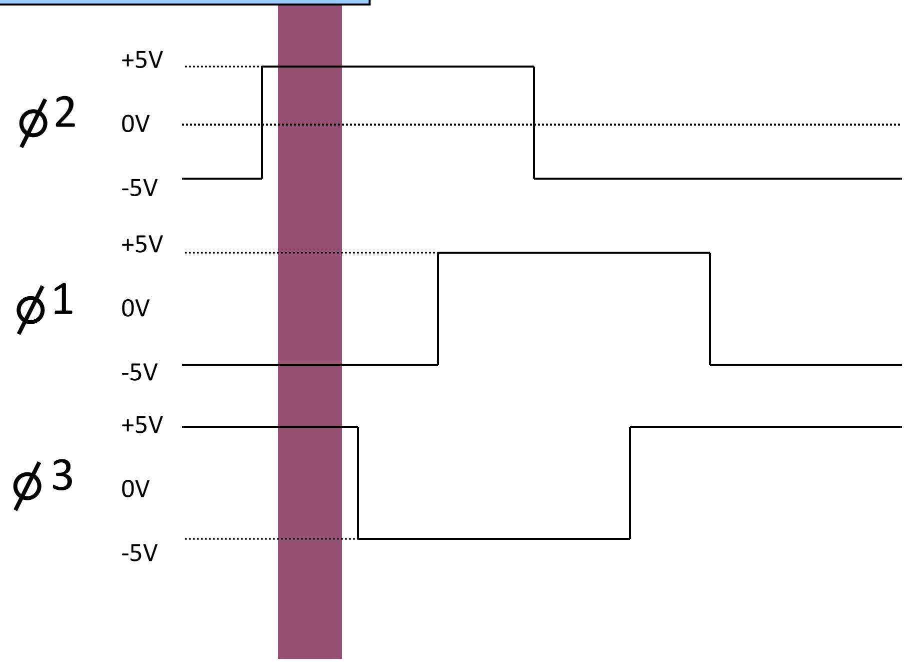


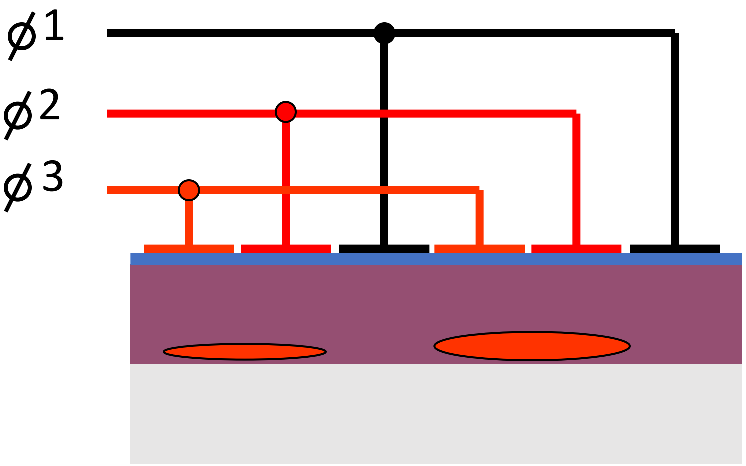
Charge Transfer in a CCD 2.



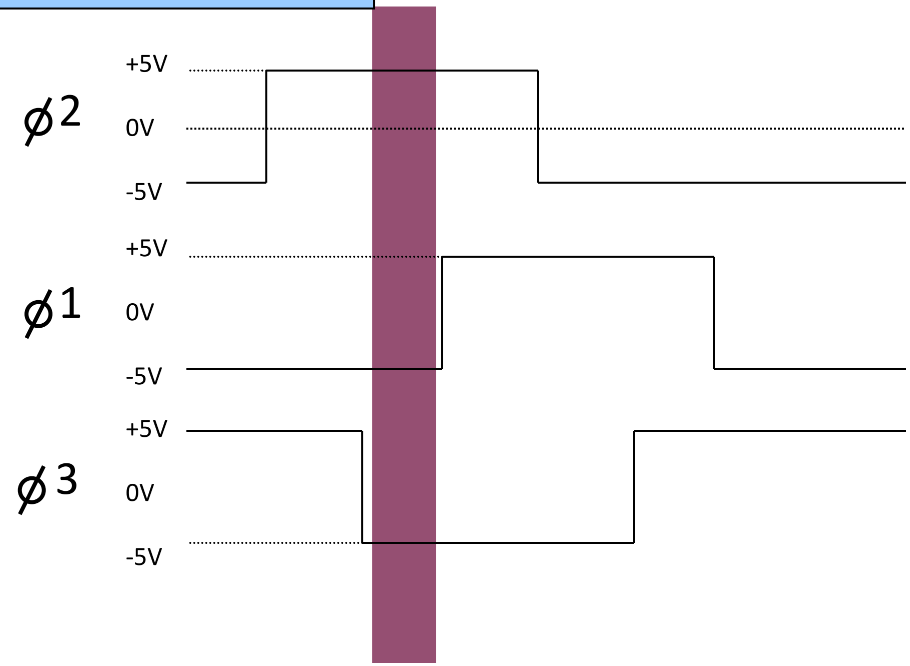


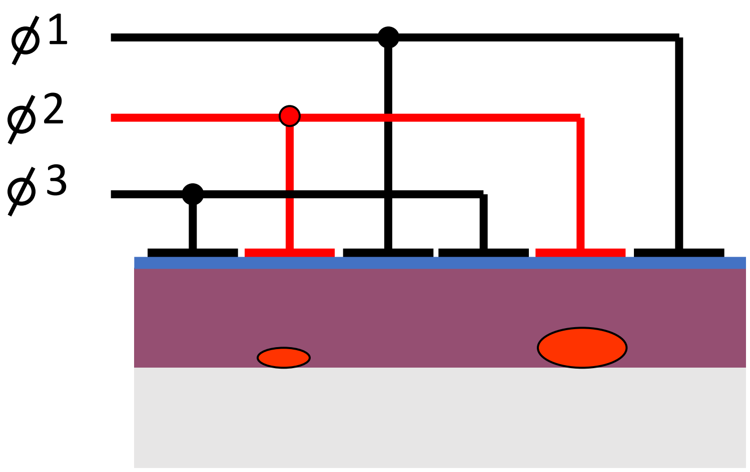
Charge Transfer in a CCD 3.



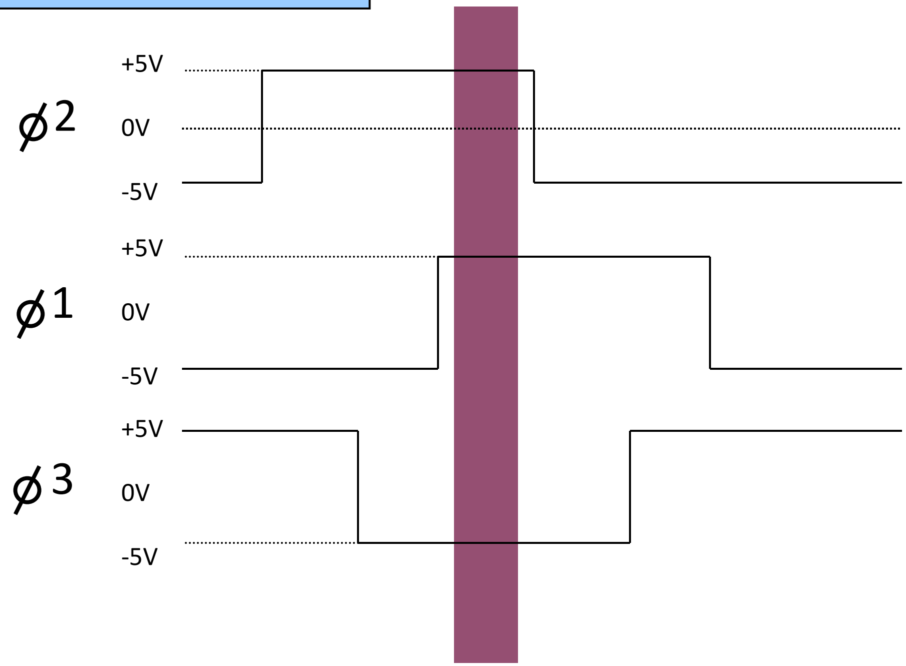


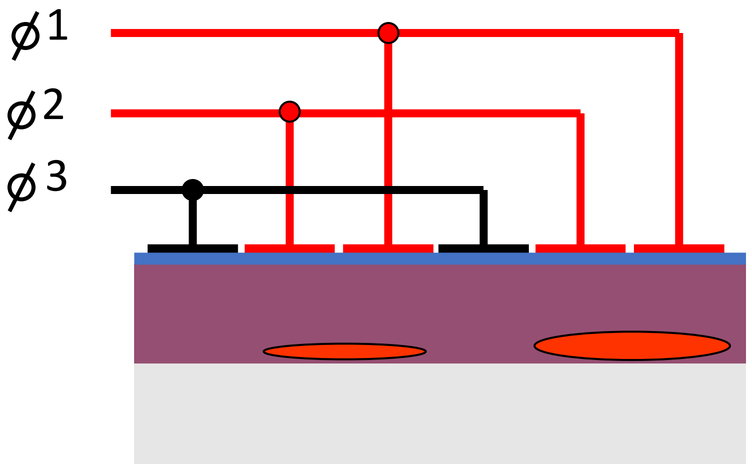
Charge Transfer in a CCD 4.



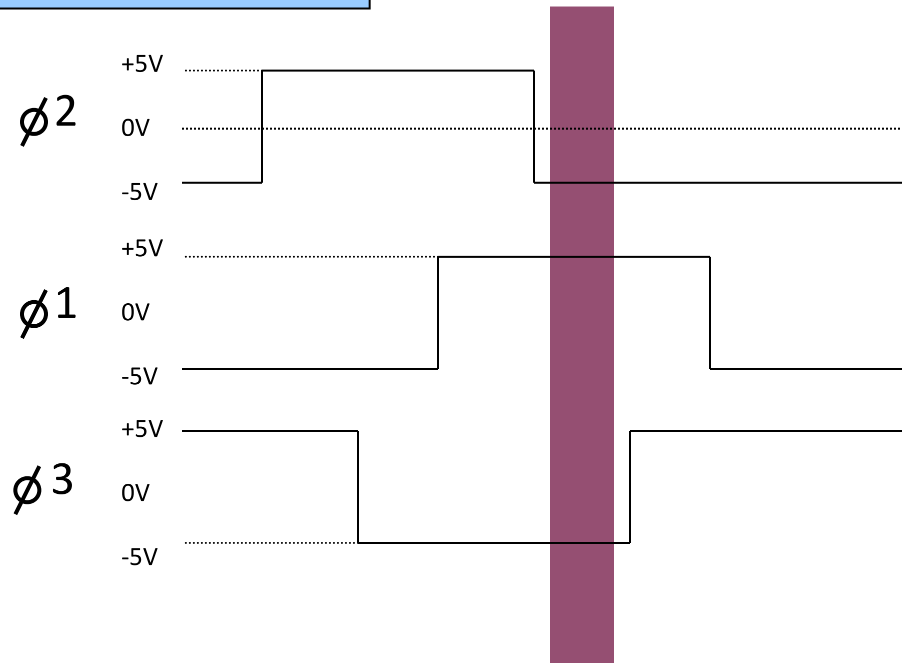


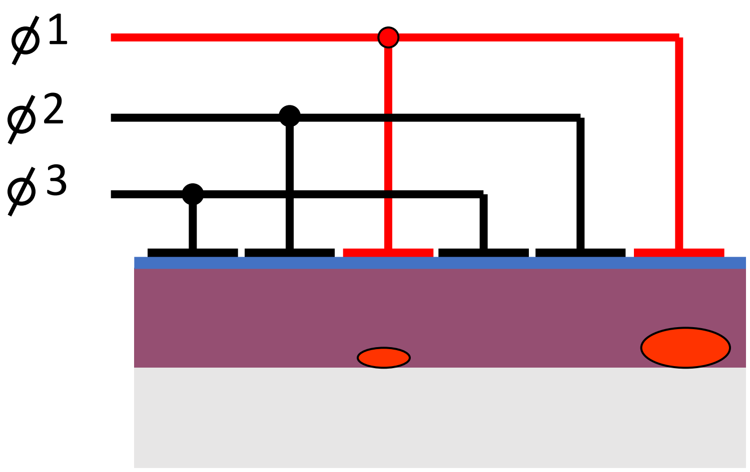
Charge Transfer in a CCD 5.





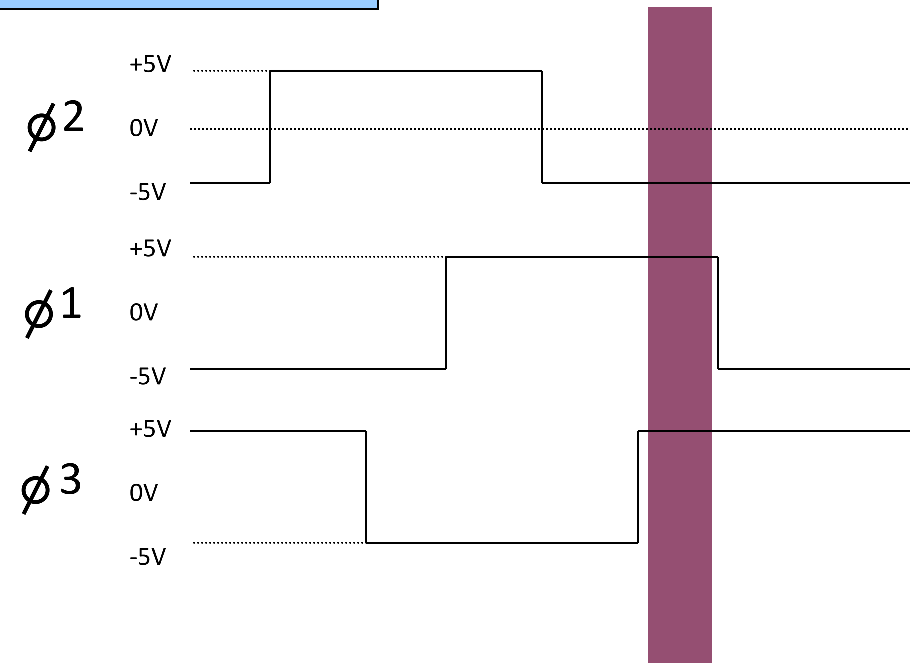
Charge Transfer in a CCD 6.

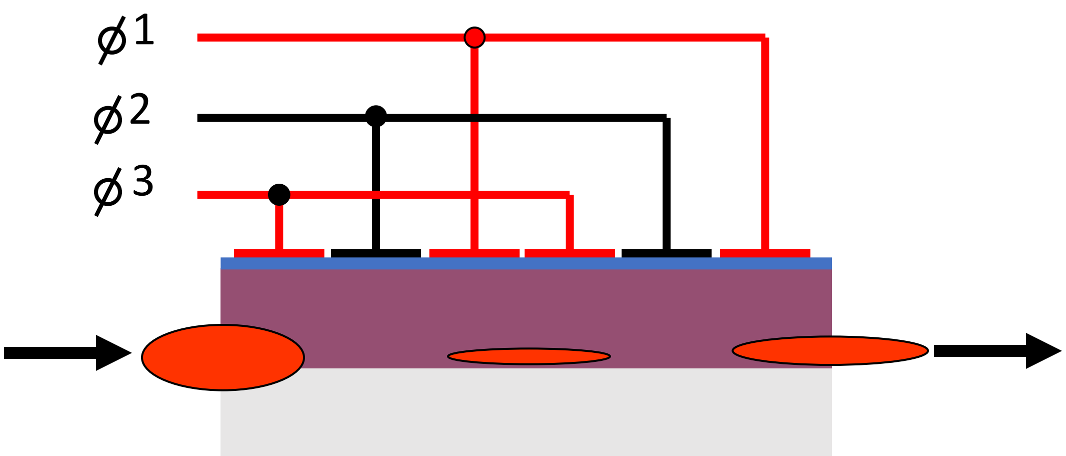




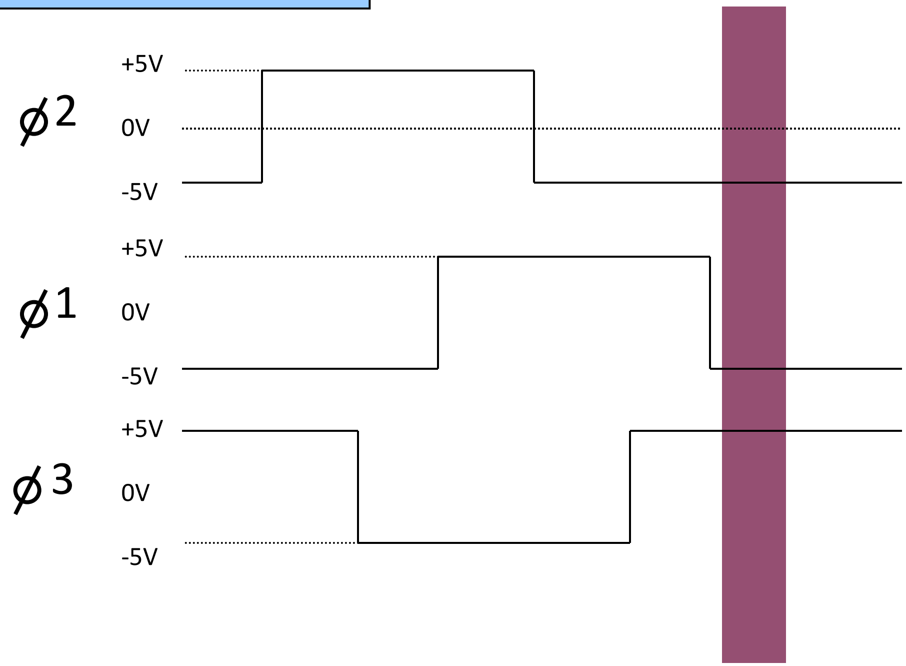
Charge Transfer in a CCD 7.

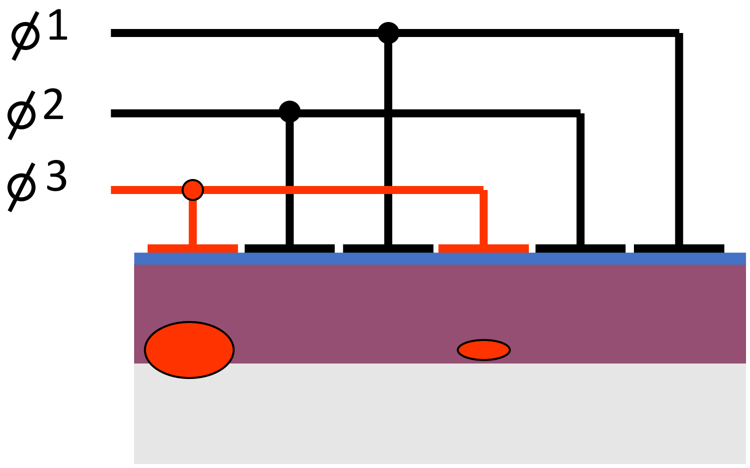
Charge packet from subsequent pixel enters from left as first pixel exits to the right.





Charge Transfer in a CCD 8.





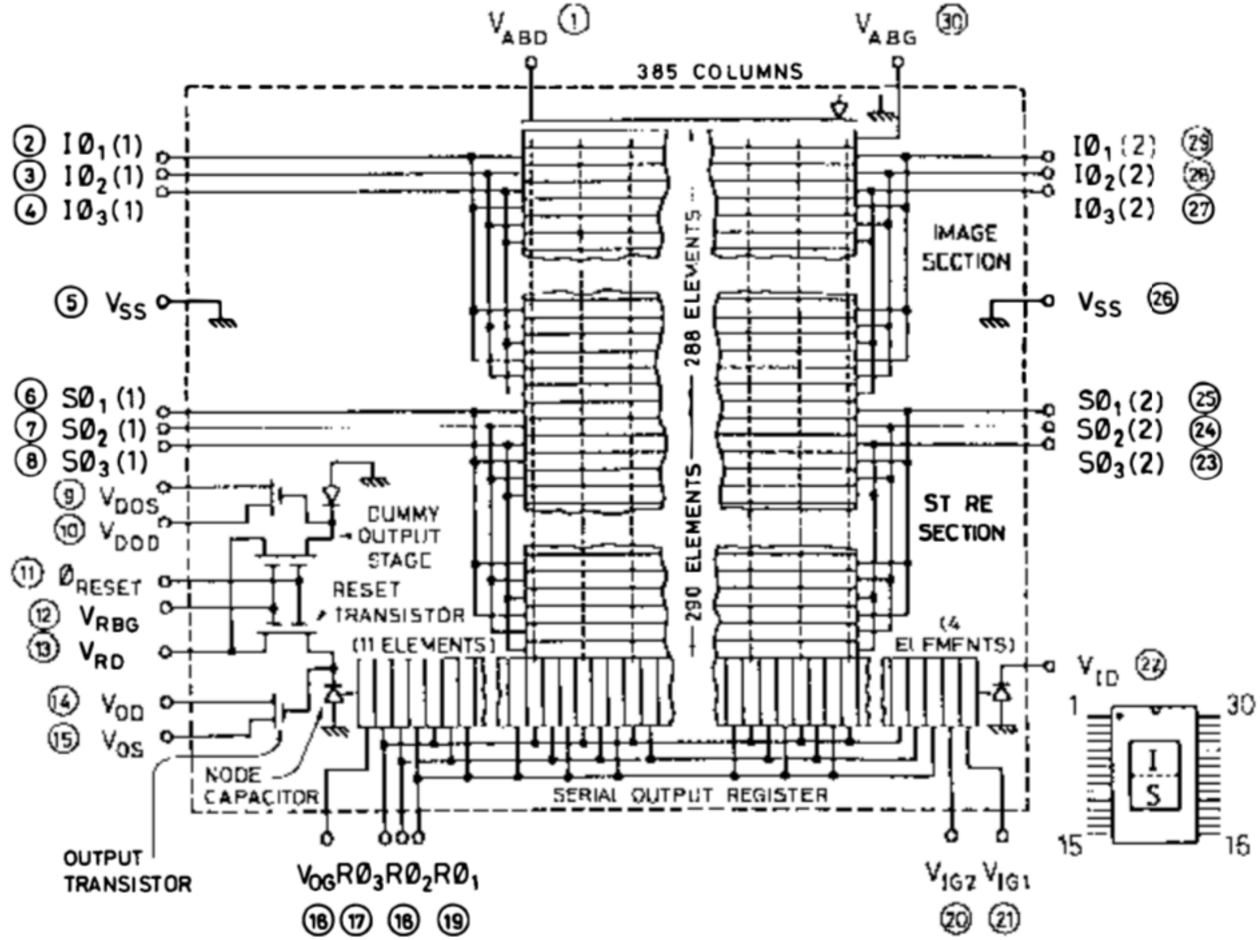


Figure 5 The full schematic of the P8600 series CCDs marketed by EEV Ltd., England. The numbers in circles indicate the device pin numbers. The detailed function of each electrode is described in the text.

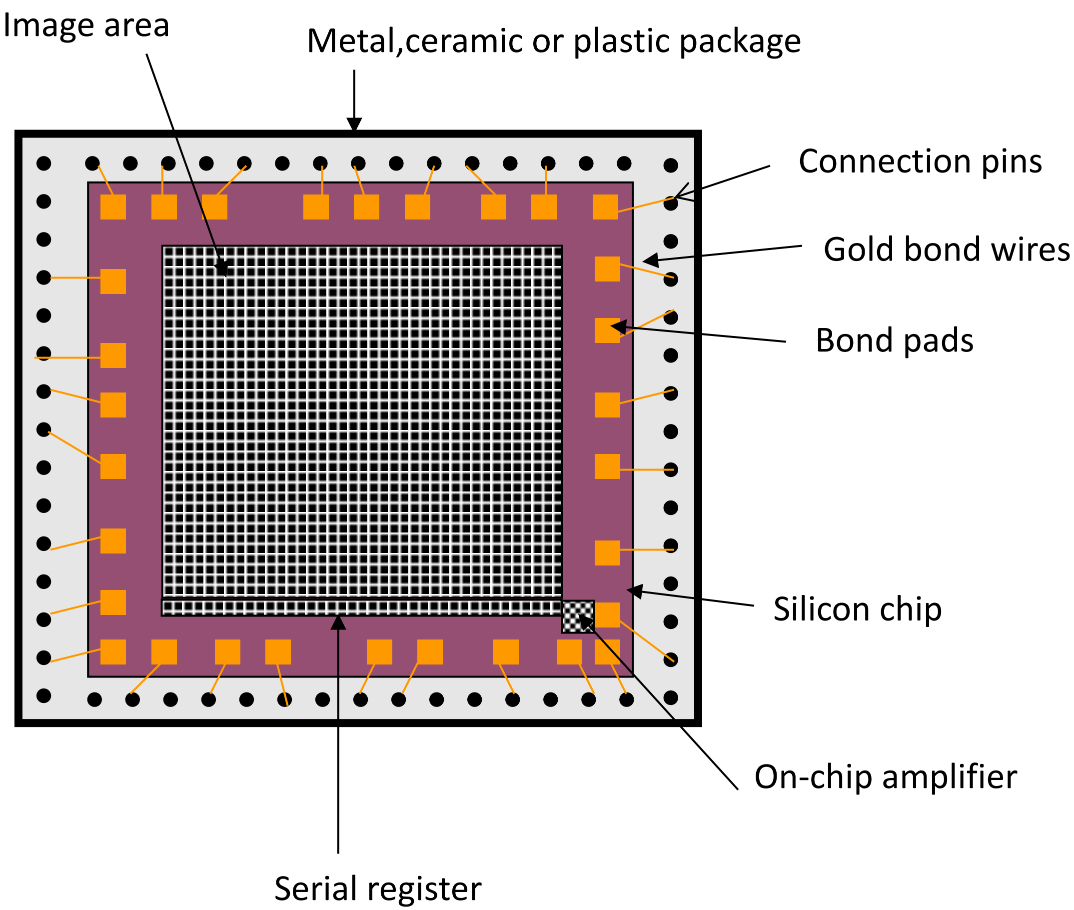
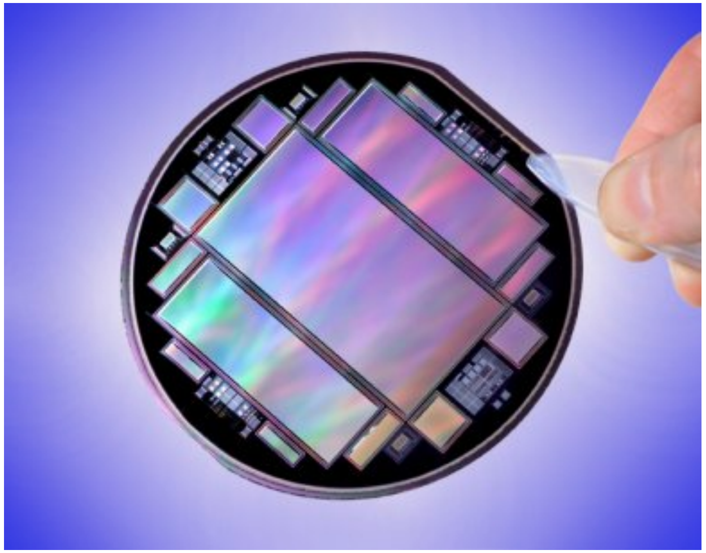


Image area
Metal,ceramic or plastic package
Connection pins
Gold bond wires
Bond pads
Silicon chip
On-chip amplifier
Serial register



Practical limitations

- All detectors have practical limitations, we'll start with some examples with CCDs

Measurement Errors

- Accuracy
  - How close a result comes to the true value
- **Precision**
  - **Closeness of multiple measurements to each other**
- **Random error**
  - **Caused by uncontrollable fluctuations in the observations (for example, photon noise)**
- Systematic error
  - Non-statistical error caused by instrument or dataset (for example bias, faulty calibration)

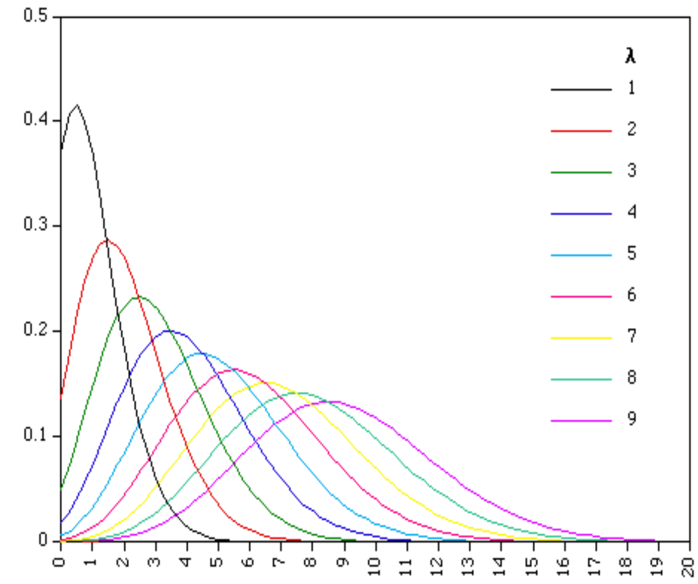
Statistics

- Statistics summarize and describe data
  - When data nicely follow a distribution, statistics are extremely useful in describing them
- Common statistical measures
  - Mean/average: $\bar{X} = \frac{1}{N}\sum_i^N X_i$
  - Median: Arrange values in order, and median is the (N/2 + 0.5) value (if N is odd) or median = $\frac{1}{2}\left(X_{\frac{N}{2}} + X_{\frac{N}{2}+1}\right)$ if N is even
  - Mode: most frequently occurring or most probable value
  - Mean Deviation: $\overline{\Delta X} = \frac{1}{N}\sum_{i=1}^N |X_i - X_{med}|$
  - Mean squares deviation: $S^2 = \frac{1}{N}\sum_{i=1}^N (X_i - \bar{X})^2$
  - Root-mean-square deviation: S
  - Order statistics (e.g. minimum, maximum, mode)

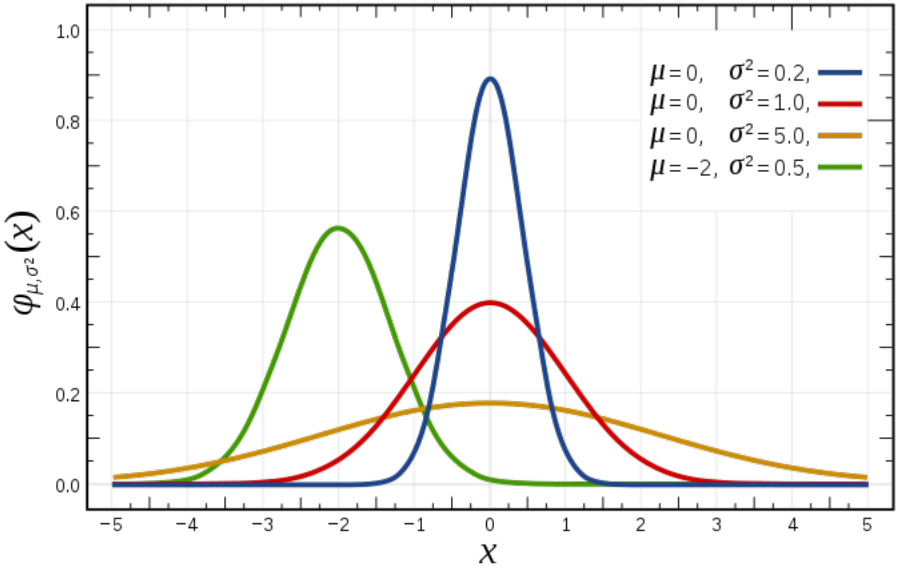
Distributions

- Distribution: expected behavior for a large number of independent measurements
- Mean and variance define the distribution function (as opposed to statistics that describe a dataset)
- Variance: $\sigma^2 \equiv \frac{1}{n}\sum(x_i - m)^2$
- Standard deviation: σ
- Poisson distribution
    - The binomial distribution in the limit of rare events (count-rate distribution)
        - Photon shot noise, radioactive decay
    - $f(x, \mu) = e^{-\mu}\mu^x / x!$
    - If μ photons are received per Δt, probability of receiving x photons in Δt is given by f(x, μ)
    - Mean: μ, variance: μ

Poisson distribution



Normal/Gaussian distribution

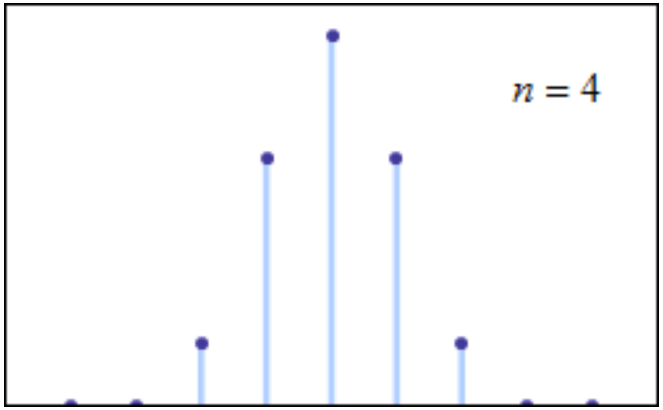


Distributions

- Gaussian (normal) distribution
  - Mean and variance are independent

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

  - Poisson distribution is indistinguishable from Gaussian distribution for large μ
- **Central limit theorem**
  - Any random sampling of values will trend towards a Gaussian distribution as N gets very large
  - $\left[\frac{M_n - \mu}{\frac{\sigma}{\sqrt{n}}}\right]$ → Gaussian distribution



https://en.wikipedia.org/wiki/File:De_moivre-laplace.gif

Detection significance

- Normal distribution gives the expected variation in background (non-signal) counts
- Detection significance is confidence in your measurement including some amount of signal
- For example, detecting a faint star

Detecting a faint star

- We point a telescope at a faint star, that our telescope cannot resolve
  - The star is a point source only covering 1 pixel
- We know that the sky's background light is emitting μ=100 photons per pixel per exposure, following Poisson statistics
- If we measure 110 photons, are we detecting the star?
- What about 150 photons?
- What can we do to increase our detection significance?

Sources of noise

- Photon noise from the star (following Poisson statistics)
  - $\sigma = \sqrt{N_*}$
- Photon noise from background sources (sky)
  - $\sigma = \sqrt{N_s}$
- Noise contributions add in quadrature
  - $\sigma_{total} = \sqrt{N_* + N_s}$
- S/N = N_* / $\sqrt{N_* + N_s}$
- What is S/N for N_* = 10, N_S = 100?
- How big does N_* need to be for S/N=100?

Error propagation

- If we have n independent estimates Xj, each with an associated error σj, then the error is estimated as the weighted mean:
  - $\overline{X_w} = \sum_{j=1}^{n} w_j \bar{X}_j / \sum_{j=1}^{n} w_j \quad , w_j = 1/\sigma_j^2$
- The variance is given by
  - $\sigma_w^2 = 1/\sum_{j=1}^{n} 1/\sigma_j^2$
- The simplest case: if errors are Gaussian and **uncorrelated**, we can just add each error source in quadrature
  - $\sigma_{total}^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2 + \sigma_d^2$

Other sources of noise

- Thermal fluctuations in CCD can also create electrons, known as the **dark current**
  - $I = Ae^{-B/kT}$
  - $\sigma = \sqrt{N_D}$, where N_D is number of dark electrons per exposure
  - Cooling down CCD reduces these thermal fluctuations
  - How would you measure the dark current?
- Readout noise
  - Usually a fixed, time-independent noise level
  - $\sigma = N_R$

CCD Signal to Noise

- For light from a star hitting a single pixel
- $\frac{S}{N} = \frac{N_*}{\sqrt{N_* + N_{sky} + N_{dark} + N_{readout}^2}}$

- If star is bright (N_* is big), S/N will scale with square root of exposure time
- If $N_{readout}$ is big, S/N will scale linearly with exposure tiime

Imaging a faint star, again

- We know our CCD has an rms readout noise of 5 counts (whenever we read it out, there is 5 counts of noise)
- We expect 5 counts per pixel per second of dark noise
  - How did we measure this?
- We expect 10 counts per pixel per second from sky background
  - How did we measure this?
- How long should we set the exposure to get a S/N of 10?

Measurement Errors

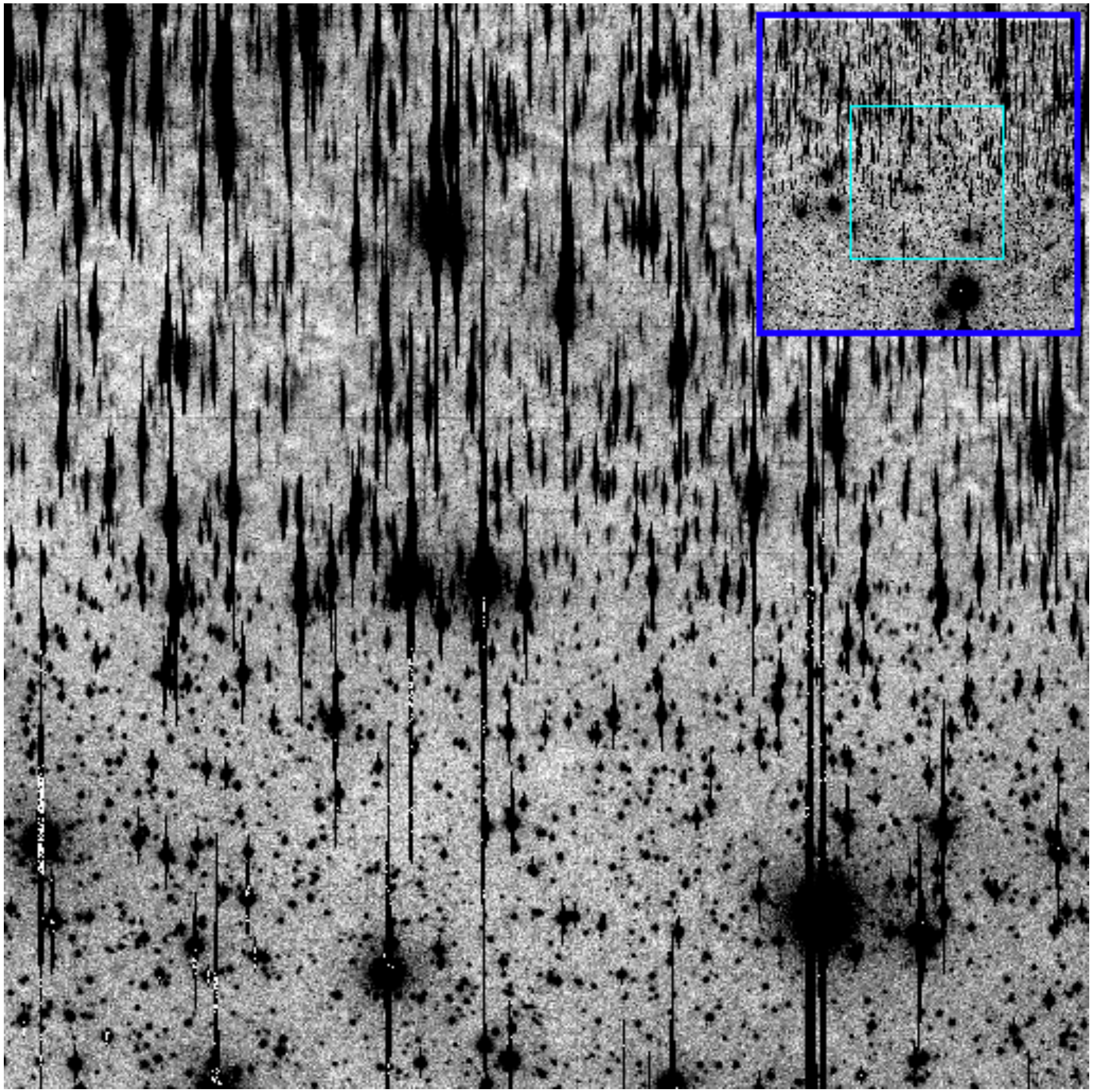
- **Accuracy**
  - **How close a result comes to the true value**
- Precision
  - Closeness of multiple measurements to each other
- Random error
  - Caused by uncontrollable fluctuations in the observations (for example, photon noise)
- **Systematic error**
  - **Non-statistical error caused by instrument or dataset (for example bias, faulty calibration)**

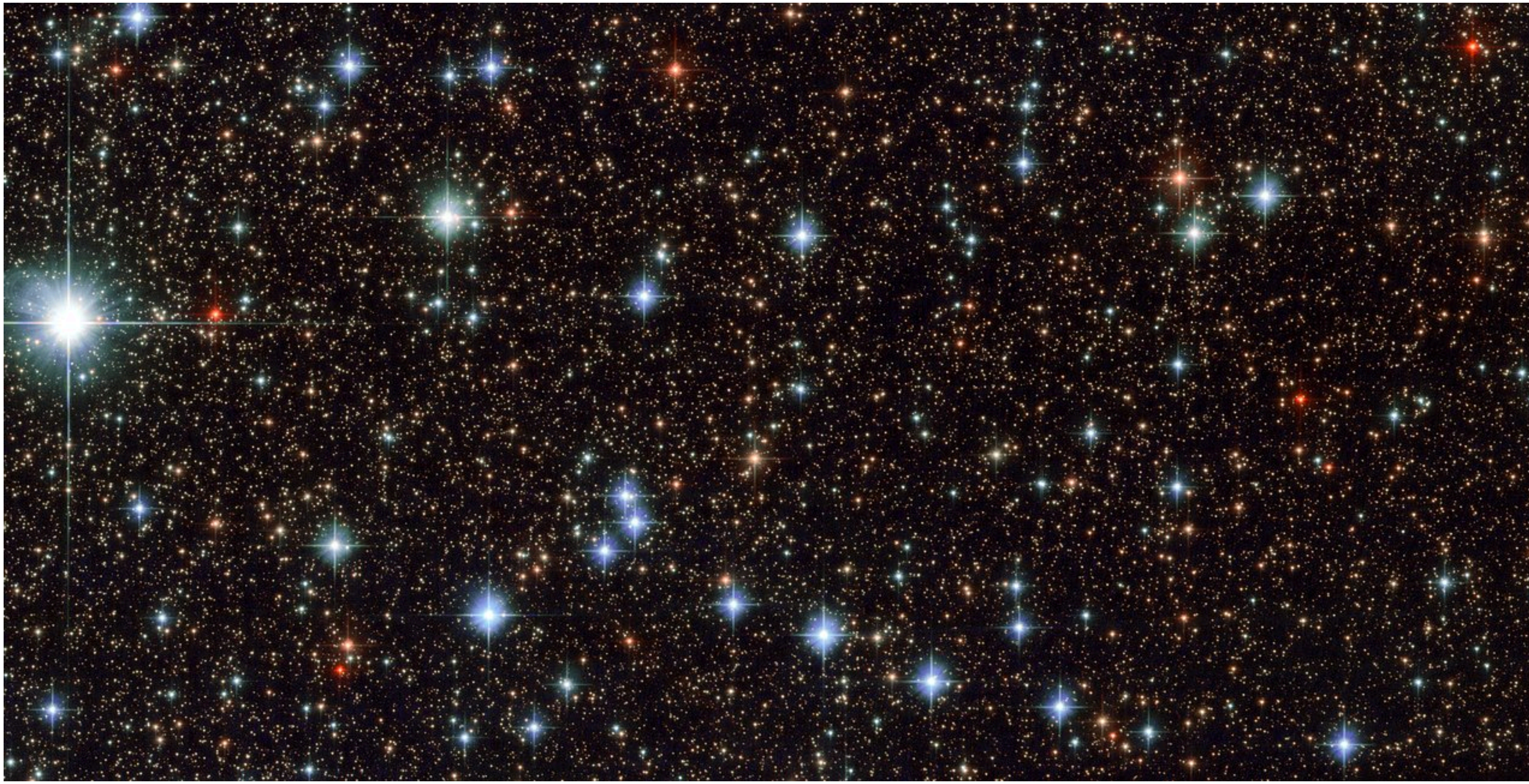
Systematic errors

- Systematic errors are not random, and can be both harder to understand and have a bigger impact on results
- Some systematic errors will never “average down”
- Reducing systematic errors typically requires understanding the experimental equipment and circumstances
- CCDs and optical telescopes have some well known and well understood systematic errors that can be accounted for in each image

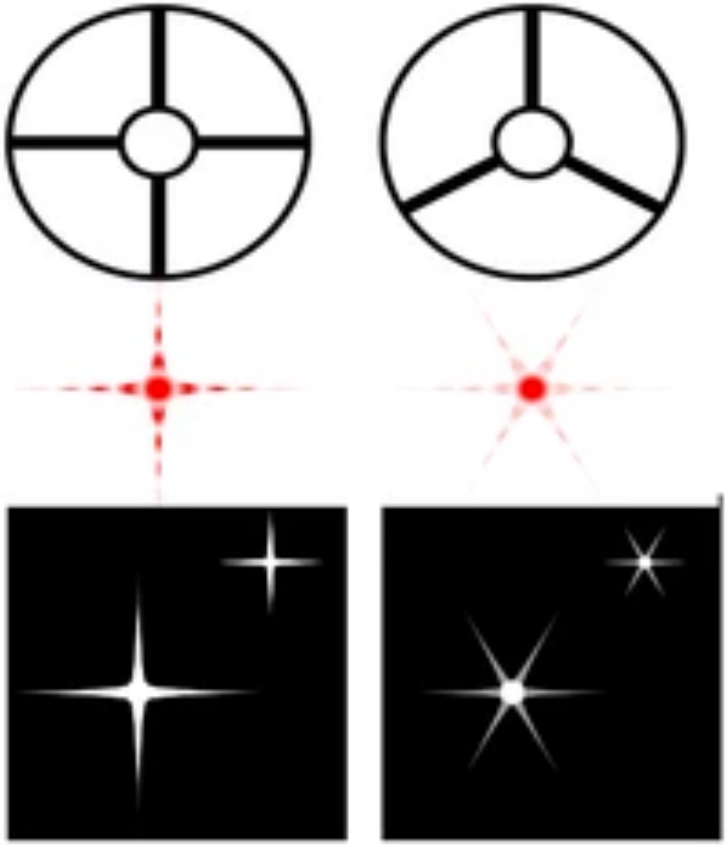
Imaging a field of stars

- We decide to use the same CCD to image a dark section of the sky, to see if we detect any stars
- We decide our cutoff for detection is 5 sigma
- In the absence of stars, we expect 1000 photons per pixel for our exposure, so we calculate that any pixel with more than 1160 photons is definitely seeing a star
- We process the data and flag any pixel that saw more than 1160 photons





Diffraction spikes



https://www.celestron.com/blogs/knowledgebase/what-is-a-diffraction-spike

Dark columns