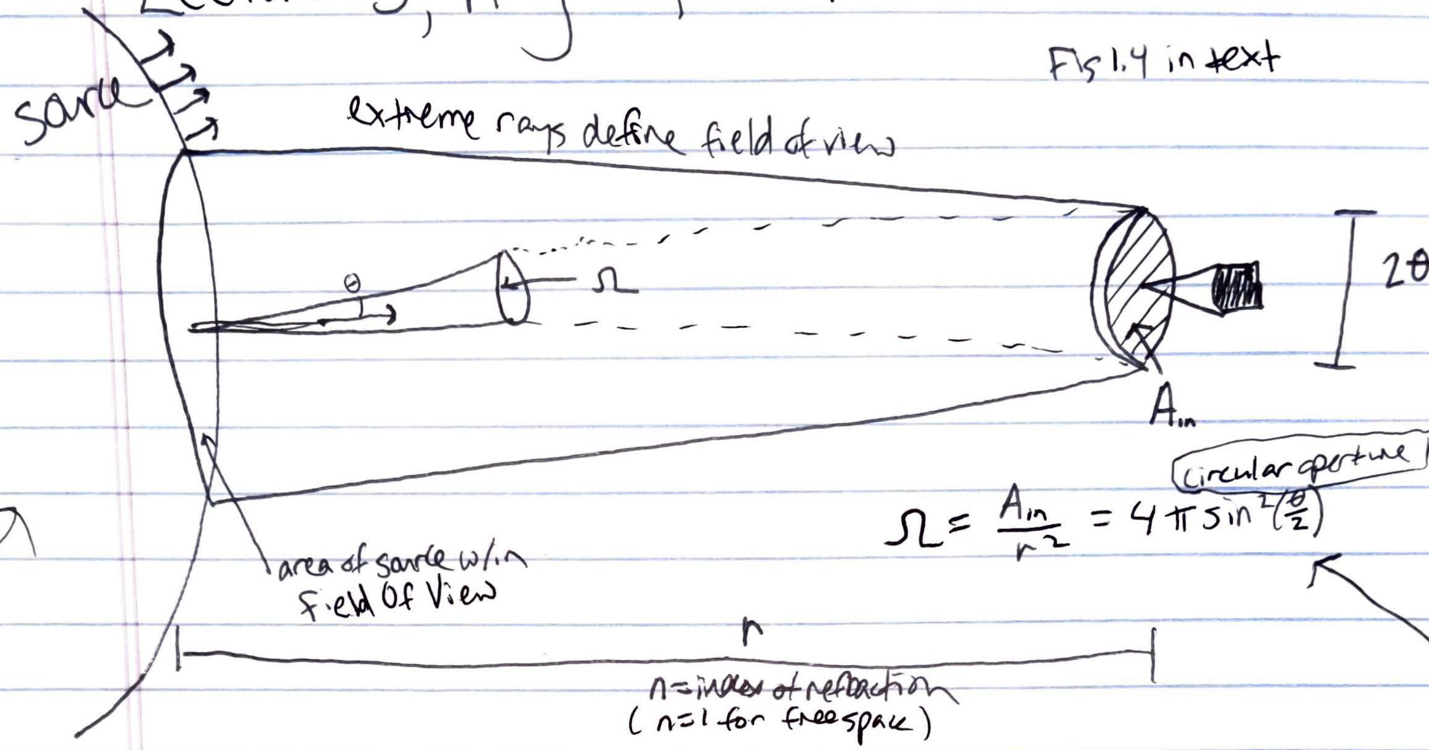
Lecture 3, Aug 26, 2019

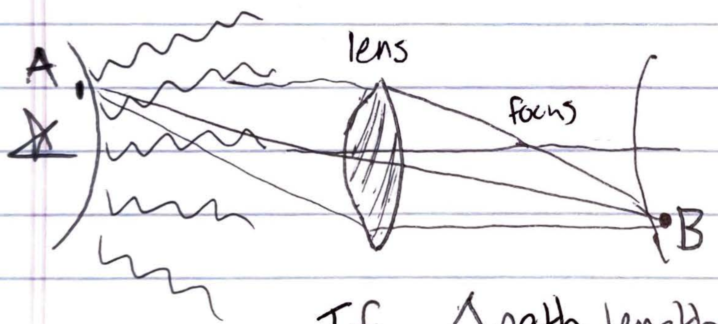
Fig 1.4 in text



$\Omega = \frac{A_{in}}{r^2} = 4\pi \sin^2\left(\frac{\theta}{2}\right)$

n = index of refraction
(n = 1 for free space)

Fermat's principle: Light always travels in the path that minimizes the time it takes to get from point A to point B



all optical paths must be the same length

To what tolerance?

If Δ path length is $\lambda/2$, destructive interference

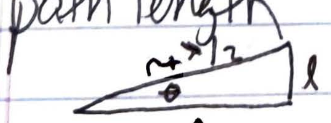
why? thermodynamics

$A\Omega$ = etendue = constant (for lossless system)
$\Omega \approx \pi\theta^2$ ($4\pi \sin^2\left(\frac{\theta}{2}\right)$ for circular aperture)

small angle approximation $\sin^2\left(\frac{\theta}{2}\right) \approx \frac{\theta^2}{4}$

θ_{max} = max resolving power
= angle over which constructive interference occurs

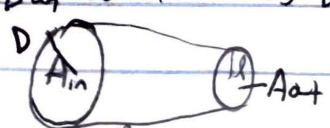
path length



$\tan\theta_{max} = \frac{\lambda}{2}$
$\approx l\,\theta_{max} = \lambda/2$

$\theta_{max} = \lambda/D$

but what limits l?

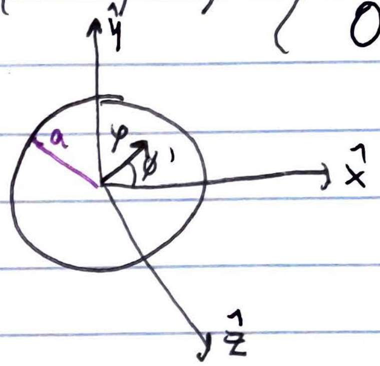


$\Omega A_{in} = \Omega A_{out}$

Diffraction limit is $\theta = \frac{\lambda}{D}$

whats $\frac{1.22\lambda}{D}$?

Uniform Illumination of Circular Aperture $\left(E(\rho, \phi') = \begin{cases} E_0 & \rho < a \\ 0 & \text{else} \end{cases} \right)$



$$I(\theta) \propto \left(\frac{J_1(2m)}{m} \right)^2$$

$$m = \pi \frac{D}{2} \theta / \lambda$$

J_1 = Bessel function of the first kind

First zero at $m = 1.916$

$\theta \cong 1.22\lambda/D$ the Rayleigh criterion

minimum separation to resolve two distinct objects

Power received by an optical system

Intensity / brightness is conserved

$A\Omega$ etendue / throughput is conserved

power $= A\Omega \cdot$ brightness

brightness of a blackbody $L_\nu = \frac{\varepsilon [2h\nu^3 n^2/c^2]}{e^{h\nu/kT} - 1}$

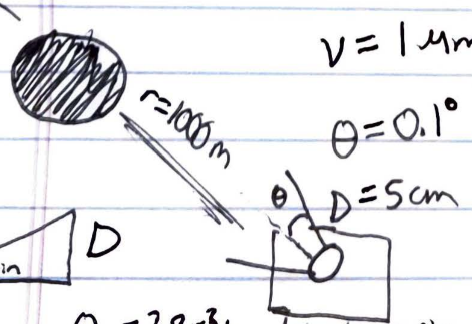
Rayleigh-Jeans Law $B(\nu, T) \simeq 2k_B \frac{\nu^2}{c^2} T_{RJ}$

when $h\nu/kT \ll 1$ (low frequencies)

$T = 1000K$

$r = 1m$

$\varepsilon = 1$

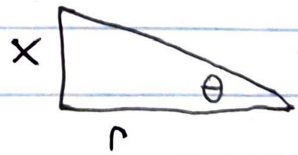


$\theta_{in} = 2.8e^{-3}$ deg detector system

$\nu = 1 \mu m$

$\theta = 0.1°$

$\Omega = 4\pi \sin^2\left(\frac{\theta}{2}\right) = \frac{A_{in}}{r^2}$



$\tan\theta = \frac{x}{r}$

$x = r\tan\theta$

$= 1000m(\tan(0.1°))$

$= (1000)(0.0017) = 1.7m$